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1-Near Mean Cordial and Biconditional Cordial Labeling of Certain Classes of Graphs

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Abstract

In this paper we prove that the graph k copies of Double star admits l-near mean Cordial labeling and biconditional Cordial labeling. Also we show that the graph $(P_2 \cup nK_1) + N_2$ is a 1-near mean Cordial graph.

AMS Subject Classification: 05C78.

Key Words and Phrases: K copies of Double star, 1-near mean Cordial graph, biconditional Cordial labeling.

1 Introduction

A graph is a collection of nodes and lines that we call vertices and edges, respectively. A graph can be labeled or unlabeled. Graph theory has rigorous applications in many fields like operations research, genetics, computer technology, physics, chemistry, communication networks, electrical network, economics and social sciences. Graph labeling is the labels which may be integers, prime numbers, modular integers, or elements of a group. A lot of research has been done in the topic of labeling of graphs. The study of graph labeling has focused on finding classes of graphs which admit a particular type of labeling. Many practical problems in real life situations have motivated the study of labeling of a graph subject to certain conditions. A systematic presentation of applications of graph labeling is given .

The concept of labeling of graphs has recently gained a lot of popularity in the area of graph theory. Most of the graph labeling method trace their origin to one introduced by Rosa [5]. A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Labeled graphs serve as useful models in a broad range of applications. Over the past five decades various labeling of graphs such as cordial labeling, prime labeling, magic labeling, antimagic labeling, bimagic labeling, mean labeling, arithmetic labeling, graceful labeling, harmonious labeling etc., have been studied extensively in the literature [2]. One of the most famous and productive labeling of graph theory is cordial labeling. This labeling was introduced by Cahit in the year 1987 [1].

In [3] Murali, Thirusangu and Madura Meenakshi, introduced the concept of Biconditional cordial labeling and proved that the path and cycle graphs are Biconditional cordial [4]. Palani, Rejila Jeya Surya, studied 1-Near mean cordial labeling of Graphs [4]. In [6], Sriram, Govindarajan, have shown that the graphs $D_2(P_n)$,$P_n(+)N_m$(when n is even), Jelly Fish $J(m,n)$ are 1-Near Mean Cordial graphs. In this paper we prove that the graph k copies of Double star admits l-near mean Cordial labeling and biconditional Cordial labeling. Also we show that the graph $(P_2 \cup nK_1) + N_2$ is a 1-near mean Cordial graph.

2 Main Results

Definition 1 (**k copies of Double star**).

k copies of Double star $K^{(1)}_{1,n,n}$, $K^{(2)}_{1,n,n}$, $\ldots$, $K^{(k)}_{1,n,n}$ then the graph $a = \langle K^{(1)}_{1,n,n} : K^{(2)}_{1,n,n} : \cdots : K^{(k)}_{1,n,n}\rangle$ is obtained by joining apex vertices of each $K^{(1)}_{1,n,n}$ and $K^{(p-1)}_{1,n,n}$ to a new vertex x_{p-1}, where $2 \leq p \leq k$, G has $2k(n+1)-1$ vertices and $2k(n+1)-2$ edges.

Definition 2 (**1-near mean Cordial Labeling**).

Let $G(V,E)$ be a simple graph. A surjective function $f : V(G) \to \{0,1,2\}$ is said to be a 1-near mean Cordial labeling if for each edge uv, the induced map

$$f^*(uv) = \begin{cases} 0; & \text{if } \frac{f(u)+f(v)}{2} \text{ is an integer} \\ 1; & \text{otherwise} \end{cases}$$

satisfies the condition $|e_f(0) - e_f(1)| \leq 1$ where $e_f(0)$ and $e_f(1)$ are the number of edges labeled by '0' and '1' respectively. G is said to be a 1-near mean Cordial graph if it admits a 1-Near mean Cordial labeling.

Definition 3 (**Biconditional Cordial Labeling**).

Let $G(V,E)$ be a graph. An surjective function $f : V \to \{0,1\}$ is said to be a biconditional Cordial labeling if for each edge uv, the induced map $f^* : E \to \{0,1\}$ is defined by,

$$f^*(uv) = \begin{cases} 1; & \text{if } f(u) = f(v) \\ 0; & \text{otherwise} \end{cases}$$

satisfies the condition $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. The graph which admits biconditional Cordial labeling is called biconditional Cordial graph.

Algorithm 4. ***Procedure (structure of k copies of Double star)***
$V \leftarrow \{\{c_1,\dots,c_k\} \cup \{v_1^1, v_2^1,\dots,v_n^1,u_1^1,\dots,u_n^1\} \cup \dots \cup \{v_1^k, v_2^k,\dots,v_n^k,u_1^k,u_1^k,\dots,u_1^k\} \cup \{x_1,x_2,\dots,x_{k-1}\}\}$
$E \leftarrow \{\{e_1^1,\dots,e_{2n}^1\} \cup \{e_1^2,\dots,e_{2n}^2\} \cup \dots \cup \{e_1^k,\dots,e_{2n}^k\} \cup \{r_1,\dots,r_{2k}\}\}$.
for $j = 1$ *to* k;
for $i = 1$ *to* n; $c_j v_i^j \leftarrow e_i^j$; $v_i^j u_i^j \leftarrow e_{n+i}^j$;
end for
end for
for $i = 1$ *to* $k-1$
$c_i x_i \leftarrow r_{2i-1}$; $c_{i+1} x_i \leftarrow r_{2i}$;
end for
end procedure

Algorithm 5. ***Input:*** ***K copies of Double star***
Procedure (1-near mean Cordial labeling for k copies of Double star)
$V \leftarrow \{\{c_1,\dots,c_k\} \cup \{v_1^1, v_2^1,\dots,v_n^1,u_1^1,\dots,u_n^1\} \cup \{v_1^2, v_2^2,\dots,v_n^2,u_1^2,\dots,u_n^2\} \cup \dots \cup \{v_1^k, v_2^k,\dots,v_n^k,u_1^k,u_1^k,\dots,u_1^k\} \cup \{x_1,x_2,\dots,x_{k-1}\}\}$
$E \leftarrow \{\{e_1^1,\dots,e_{2n}^1\} \cup \{e_1^2,\dots,e_{2n}^2\} \cup \dots \cup \{e_1^k,\dots,e_{2n}^k\} \cup \{r_1,\dots,r_{2k}\}\}$.
for $j = 1$ *to* k;
if $j \equiv 1 \ (mod\ 2)$
$f(c_j) \leftarrow 1$
else
$f(c_j) \leftarrow 2$
end for
for $j = 1$ *to* k
if $j \equiv 1 \ (mod\ 2)$
for $i = 2$ *to* n

$$f(v_i^j) \leftarrow \begin{cases} 1; & i \equiv 0 \ (mod\ 2) \\ 2; & i \equiv 1 \ (mod\ 2) \end{cases};$$

$$f(u_i^j) \leftarrow \begin{cases} 0; & i \equiv 0 \ (mod\ 2) \\ 2; & i \equiv 2 \ (mod\ 2) \end{cases};$$

$f(v_1^j) \leftarrow 0$; $f(u_1^j) \leftarrow 0$;
end for
else
for $i = 2$ *to* n

$$f(v_i^j) \leftarrow \begin{cases} 1; & i \equiv 0 \ (mod\ 2) \\ 2; & i \equiv 1 \ (mod\ 2) \end{cases};$$

$$f(u_i^j) \leftarrow \begin{cases} 2; & i \equiv 0 \ (mod\ 2) \\ 0; & i \equiv 1 \ (mod\ 2) \end{cases};$$

$f(v_1^j) \leftarrow 0$; $f(u_1^j) \leftarrow 0$;
end for
end if
end for
for $i = 1$ *to* $k-1$
$f(x_i) \leftarrow 1$;

end for
end procedure
Output: *labeled K copies of Double star.*

Theorem 6. *K copies of Double star admits 1-Near Mean Cordial labeling.*

Proof. From the construction of k copies of Double star, $\langle K_{1,n,n}^{(1)} : K_{1,n,n}^{(2)} : \cdots : K_{1,n,n}^{(k)}\rangle$ has $2k(n+1)-1$ vertices and $2k(n+1)-2$ edges. Denote the vertex set and edge set using algorithm 4. To prove k copies of Double star admits 1-Near mean Cordial labeling, we have to show that there exists a function $f : V \to \{0,1,2\}$ as given in algorithm 5. In order to get the edge labels, we define a map $f^* : E \to \{0,1\}$ such that for each edge $uv \in E$, $f^*(uv) = \begin{cases} 0; & \text{if } \frac{f(v)+f(v)}{2} \text{ is an integer} \\ 1; & \text{otherwise.} \end{cases}$.
Thus the edge labels are as follows:

(i) for $j = 1$ to k and for $i = 1$ to n

$$f^*(v_i^j u_i^j) = f^*(c_j v_i^j) = \begin{cases} 0; & i \equiv 1 \pmod 2; j \equiv 1 \pmod 2 \\ 1; & i \equiv 0 \pmod 2 \end{cases};$$

$$f^*(v_i^j u_i^j) = f^*(c_j v_i^j) = \begin{cases} 1; & i \equiv 0 \pmod 2; j \equiv 0 \pmod 2 \\ 0; & i \equiv 1 \pmod 2 \end{cases};$$

(ii) for $i = 1$ to $k-1$

$$f^*(c_i x_i) = \begin{cases} 1; & i \equiv 1 \pmod 2 \\ 0; & i \equiv 0 \pmod 2 \end{cases}; \quad f^*(c_{i+1} x_i) = \begin{cases} 0; & i \equiv 1 \pmod 2 \\ 1; & i \equiv 0 \pmod 2 \end{cases};$$

Clearly, the number of edges labeled by '0' and '1' is $k(n+1)-1$. Thus the number of edges labeled '0' and '1' are differ by atmost 1. Hence the K copies of Double star admits 1-Near mean Cordial labeling. □

Example 7. *2 copies of Double star is 1-near mean Cordial.*

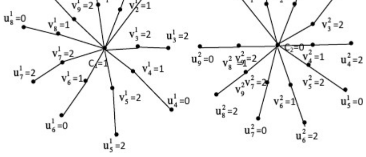

Figure 1: 1-Near Mean Cordial labeling for 2 copies of Double star $\langle K_{1,9,9}^{(1)} : K_{1,9,9}^{(2)}\rangle$

Algorithm 8. ***Input: (k copies of Double star)***
Procedure (biconditional Cordial labeling for k copies of Double star)
$V \leftarrow \{\{c_1, \ldots, c_k\} \cup \{v_1^1, v_2^1, \ldots, v_n^1, u_1^1, \ldots, u_n^1\} \cup \{v_1^2, v_2^2, \ldots, v_n^2, u_1^2, \ldots, u_n^2\} \cup \cdots \cup \{v_1^k, v_2^k, \ldots, v_n^k, u_1^k, u_1^k, \ldots, u_1^k\} \cup \{x_1, x_2, \ldots, x_{k-1}\}\}$
$E \leftarrow \{\{e_1^1, \ldots, e_{2n}^1\} \cup \{e_1^2, \ldots, e_{2n}^2\} \cup \cdots \cup \{e_1^k, \ldots, e_{2n}^k\} \cup \{r_1, \ldots, r_{2k}\}\}$.
for $j = 1$ *to* k;
  if $j \equiv 1 \ (mod\ 2)$
    for $i = 1$ *to* n
      $f(v_i^j) \leftarrow \begin{cases} 1; & i \equiv 1, 0 \ (mod\ 4) \\ 0; & i \equiv 2, 3 \ (mod\ 4) \end{cases}$
      $f(c_j) \leftarrow 1$
    else
      $f(v_i^j) \leftarrow \begin{cases} 0; & i \equiv 0, 1 \ (mod\ 4) \\ 1; & i \equiv 2, 3 \ (mod\ 4) \end{cases}$
      $f(c_j) \leftarrow 0$
    end for
  end if
end for
if $n \equiv 3 \ (mod\ 4)$
  for $j = 1$ *to* k
    if $j \equiv 1 \ (mod\ 2)$
      $f(u_i^j) \leftarrow \begin{cases} 1; & i \equiv 1, 2 \ (mod\ 4) \\ 0; & i \equiv 0, 3 \ (mod\ 4) \end{cases}$
    else
      $f(u_i^j) \leftarrow \begin{cases} 0; & i \equiv 1, 2 \ (mod\ 4) \\ 1; & i \equiv 0, 3 \ (mod\ 4) \end{cases}$
    end if
  end for
else if $n \equiv 1 \ (mod\ 4)$
  for $j = 1$ *to* k
    $f(u_i^j) \leftarrow \begin{cases} 0; & i \equiv 1, 2 \ (mod\ 4) \\ 1; & i \equiv 0, 3 \ (mod\ 4) \end{cases}$
  end for
end if
if $n \equiv 0 \ (mod\ 2)$
  for $j = 1$ *to* k
    if $j \equiv 1 \ (mod\ 2)$
      $f(u_i^j) \leftarrow \begin{cases} 1; & i \equiv 1, 2 \ (mod\ 4) \\ 0; & i \equiv 0, 3 \ (mod\ 4) \end{cases}$
    else
      $f(u_i^j) \leftarrow \begin{cases} 0; & i \equiv 1, 2 \ (mod\ 4) \\ 1; & i \equiv 0, 3 \ (mod\ 4) \end{cases}$
    end if
  end for

end if
if $n \equiv 2\ (mod\ 4)$
for $i = 1$ *ot* $k - 1$
$$f(x_i) \leftarrow \begin{cases} 1; & i \equiv 1, 2\ (mod\ 4) \\ 0; & i \equiv 0, 3\ (mod\ 4) \end{cases}$$
else
$$f(x_i) \leftarrow \begin{cases} 0; & i \equiv 1\ (mod\ 4) \\ 1; & i \equiv 0\ (mod\ 4) \end{cases}$$
end for
end if
end procedure
Output: *k copies of Double star admits biconditional Cordial labeling.*

Theorem 9. *k copies of Double star admits biconditional Cordial labeling*

Proof. From the construction of k copies of Double star we have $2k(n + 1) - 1$ vertices and $2k(n+1)-2$ edges. Denote the vertex set and edge set using algorithm 8.

To prove k copies of Double star admits biconditional Cordial labeling, we have to show that there exists a function $f : V \to \{0, 1\}$ such that the induced map $f^* : E \to \{0, 1\}$ is defined by

$$f^*(uv) = \begin{cases} 1; & f(u) = f(v) \\ 0; & \text{otherwise} \end{cases}$$

satisfies the property that $|V_f(0) - V_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Case (i)

When $n \equiv 1 \pmod 2$ and $n \equiv 0 \pmod 4$

(i) for even k, the number of vertices labeled by '0' is $k(n+1)$ and '1' is $k(n+1)-1$.

(ii) For odd k, the number of vertices labeled by '0' is $k(n + 1) - 1$ and '1' is $k(n + 1)$.

Case (ii)

When $n \equiv 2 \pmod 4$. The number of vertices labeled by '1' is $k(n + 1)$ and '0' is $k(n + 1) - 1$. From the above cases, the number of vertices labeled by '0' and '1' are differ by atmost 1.

In order to get the edge labels, we define a map $f^* : E \to \{0, 1\}$ as follows:

(i) For $j = 1$ to k, for $i = 1$ to n

$$f^*(c_j v_i^j) = \begin{cases} 1; & i \equiv 1, 0 \pmod 4 \\ 0; & i \equiv 2, 3 \pmod 4 \end{cases}$$

$$f^*(v_i^j u_i^j) = \begin{cases} 1; & i \equiv 1 \pmod 2 \\ 0; & i \equiv 0 \pmod 2 \end{cases} ; n \equiv 3 \pmod 4$$

$$f^*(v_i^j u_i^j) = \begin{cases} 0; & i \equiv 1 \pmod 2 \\ 1; & i \equiv 0 \pmod 2 \end{cases}; \ n \equiv 0, 2, 1 \pmod 4$$

(ii) For $i = 1$ to $k - 1$

$$f^*(c_i x_i) = \begin{cases} 1; & i \equiv 1, 0 \pmod 4 \\ 0; & i \equiv 2, 3 \pmod 4 \end{cases}; \ n \equiv 2 \pmod 4$$

$$f^*(c_{i+1} x_i) = \begin{cases} 0; & i \equiv 1, 0 \pmod 4 \\ 0; & i \equiv 2, 3 \pmod 4 \end{cases}; \ n \equiv 2 \pmod 4$$

$$f^*(c_i x_i) = \begin{cases} 0; & i \equiv 1 \pmod 2 \\ 1; & i \equiv 0 \pmod 2 \end{cases}; \ n \equiv 0, 1, 3 \pmod 4$$

$$f^*(c_{i+1} x_i) = \begin{cases} 0; & i \equiv 1 \pmod 2 \\ 1; & i \equiv 0 \pmod 2 \end{cases}; \ n \equiv 0, 1, 3 \pmod 4$$

Under this map, the number of edges labeled by '0' and '1' $k(n+1) - 1$. Thus the number of edges labeled by '0' and '1' are differ by atmost 1. Hence k copies of Double star admits biconditional Cordial labeling. □

Example 10. *The Biconditional cordial labeling for 2 copies of Double star* $\langle K_{1,9,9}^{(1)} : K_{1,9,9}^{(2)} \rangle$ *is given in figure 2.*

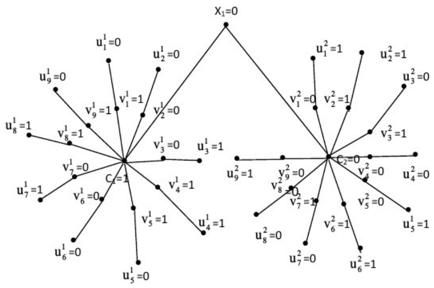

Figure 2: Biconditional Cordial labeling for 2 copies of Double star $(K_{1,9,9}^{(1)}; K_{1,9,9}^{(2)})$

Definition 11 $((P_2 \cup nK_1) + N_2)$**.**
The graph $(P_2 \cup nK_1) + N_2$ is a graph with vertex set $V = \{v_2, u_1, u_2 \ldots u_{n+1}\} \cup \{v_1, v_3\}$ and edge set $\{[(v_1u_1), (v_1v_2), (v_3u_1), (v_3v_2), (u_1v_2)] \cup [(v_1u_i) \cup (v_3u_i) : 2 \leq i \leq n+1]\}$.

Algorithm 12. *Procedure (Structure of* $(P_2 \cup nK_1) + N_2$

$V \leftarrow \{v_1, v_2, v_3 \cup u_1, \ldots, u_{n+1}\}$
$E \leftarrow \{e_1, e_2, e_3 \cup e'_1, \ldots, e'_{2(n+1)}\}$
  $v_1v_2 \leftarrow e_1$;
  $v_2v_3 \leftarrow e_2$;
  $v_2u_1 \leftarrow e_3$;
  for $i = 1$ *to* $n+1$
    $v_1u_i \leftarrow e'_{2i-1}$;
    $v_3u_i \leftarrow e'_{2i}$;
  end for
  end procedure

Algorithm 13. ***Input:*** $P_2 \cup nK_1 + N_2$

Procedure (1-Near mean Cordial labeling for $(P_2 \cup nK_1) + N_2$*)*
$V \leftarrow \{v_1, v_2, v_3 \cup u_1, \ldots, u_{n+1}\}$
$E \leftarrow \{e_1, e_2, e_3 \cup e'_1, \ldots, e'_{2(n+1)}\}$
  $f(v_1) \leftarrow 1$;
  $f(v_2) \leftarrow 0$;
  $f(v_3) \leftarrow 2$;
  for $i = 1$ *to* $n+1$

$$f(u_i) \leftarrow \begin{cases} 0; & i \equiv 1 \ (mod\ 4) \\ 2; & i \equiv 2 \ (mod\ 4) \\ 1; & i \equiv 0, 3 \ (mod\ 4) \end{cases};$$

  end for
end procedure
Output: $(P_2 \cup nK_1) + N_2$ *admits 1-Near mean Cordial labeling.*

Theorem 14. *The graph* $(P_2 \cup nk_1) + N_2$ *is 1-Near mean Cordial graph.*

Proof. From the construction of $(P_2 \cup nk_1) + N_2$, we have $n+4$ vertices and $2(n+1)+3$ edges. Denote the vertex set and edge set using algorithm 13.

To prove $(P_2 \cup nk_1)+N_2$ is 1-Near mean Cordial graph, we have to show that there exists a function $f : V \rightarrow \{0,1,2\}$ such that there is an induced map $f^* : E \rightarrow \{0,1\}$ is defined by

$$f^*(uv) = \begin{cases} 0; & \text{if } \frac{f(u)+f(v)}{2} \text{ is an integer} \\ 1; & \text{otherwise} \end{cases}$$

In order to get edge labels, we define a map $f^* : E \rightarrow (0,1)$ as follows:

(i) for $i = 1$ to $n+1$

$$f^*(v_1u_i) = \begin{cases} 0; & i \equiv 1,2 \pmod 4 \\ 1; & i \equiv 0,3 \pmod 4 \end{cases}; \quad f^*(v_3u_i) = \begin{cases} 1; & i \equiv 1,2 \pmod 4 \\ 0; & i \equiv 0,3 \pmod 4 \end{cases}$$

(ii) $f^*(v_1v_2) = 0,\ f^*(v_2v_3) = f^*(v_2u_1) = 1$

Under this map, the number of edges labeled by '0' and '1' are $n+2$ and $n+3$ respectively. Thus the number of edges labeled by '0' and '1' are differ by atmost 1. Hence $(P_2 \cup nK_1) + N_2$ is 1-Near Mean Cordial graph □

Example 15. *1-Near Mean labeling for* $(P_2 \cup 5K_1) + N_2$ *is shown in figure 3.*

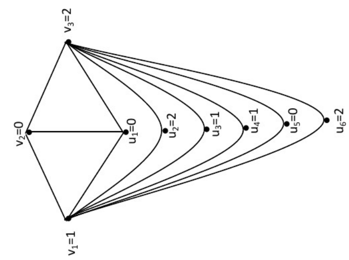

Figure 3: 1-Near Mean labeling for the graph $(P_2 \cup 5K_1) + N_2$

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