

Generators in Intuitionistic Topological Spaces

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Abstract

The aim of this paper is to show that the possible number of intuitionistic sets for a nonempty set with 'n' elements. Also a new class of sets called intuitionistic generator and intuitionistic prime generator are defined and using it intuitionistic generator topology and intuitionistic prime generator topology are formed and their special properties are studied.

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1 Introduction

Coker [1] introduced the concept of intuitionistic sets in 1996. This is a discrete form of intuitionistic fuzzy set, where all the sets are entirely the crisp sets. This concept gives us more flexible approaches to represent vagueness in mathematical objects including engineering fields with classical set logic. Further, he defined operations and functions on intuitionistic sets analogous to set theory. Dogan Coker [2], in 2000, introduced intuitionistic topological spaces based on intuitionistic sets which is a classical version of the concept in intuitionistic fuzzy topological spaces.

In this paper, the maximum number of intuitionistic sets for a nonempty set with 'n' elements is calculated and hence shown that the general topology becomes a sub category of intuitionistic topological space. Also, a new class of sets called intuitionistic generator and intuitionistic prime generator are defined and using it intuitionistic generator topology and intuitionistic prime generator topology are formed and their special properties are studied. It is proved that the intuitionistic generator topology is always extremally disconnected and the intuitionistic open sets of the generator topology are intuitionistic closed.

2 Preliminaries

Throughout the present study, a space X means an intuitionistic topological space (X, τ) unless otherwise mentioned.

Definition 2.1. [1] Let X be a nonempty set. An intuitionistic set (IS for short) A is an object having the form $A = \langle X, A_1, A_2 \rangle$, where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \phi$. The set A_1 is called the set of members of A , while A_2 is called the set of non-members of A .

Definition 2.2. [2] An intuitionistic topology (IT for short) on a nonempty set X is a family τ of IS's in X satisfying the following axioms:

(i) $\phi, \tilde{X} \in \tau$,

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(iii) $\cup G_i \in \tau$ for any arbitrary family $\{G_i : i \in J\} \subseteq \tau$.

In this case, the pair (X, τ) is called an intuitionistic topological space (ITS for short) and any intuitionistic set in τ is known as an intuitionistic open set (IOS for short) in X .

Definition 2.3. [1] Let (X, τ) be an ITS and $A = \langle X, A_1, A_2 \rangle$ be an IS in X . Then the interior and closure of A are defined as

$Icl(A) = \bigcap \{K : K \text{ is an ICS in } X \text{ and } A \subseteq K\}$

$Iint(A) = \bigcup \{G : G \text{ is an IOS in } X \text{ and } G \subseteq A\}$

Definition 2.4. [4] If there exists an intuitionistic regular open set A in X such that $\phi \neq A \neq \tilde{X}$, then X is called super disconnected. X is called super connected, if X is not super disconnected.

Definition 2.5. [4] An intuitionistic topological space X is called C_5 -disconnected if there exists an intuitionistic set A which is both intuitionistic open and intuitionistic closed such that $\phi \neq A \neq \tilde{X}$. X is called C_5 -connected, if X is not C_5 -disconnected.

Definition 2.6. An intuitionistic topological space (X, τ_G) is said to be IG-extremally disconnected if $Icl(B) \in IOS(X)$ for every $B \in IOS(X)$.

3 Possible number of intuitionistic sets of a nonempty set

As intuitionistic sets can be viewed as a generalization of sets, in this section binomial theorem is used to prove that the possible number of intuitionistic sets for a nonempty set with 'n' elements is 3^n .

Theorem 3.1. The possible number of intuitionistic sets of a nonempty set with 'n' elements is 3^n .

Proof. Let X be a nonempty set with 'n' elements. Then the intuitionistic sets of X are of the form $\langle X, A_1, A_2 \rangle$. Let p represents the number of elements that belong to A_1 , q represents the number of elements that belong to A_2 and the values present inside the cells represent the number of intuitionistic sets of the form $\langle X, A_1, A_2 \rangle$ with respect to the numbers in p and q . And S_i

$p \setminus q$	0	1	2	3	4	—	—	n-1	n
0	$\binom{n}{0}$	$\binom{n}{1}\binom{n-1}{0}$	$\binom{n}{2}\binom{n-2}{0}$	$\binom{n}{3}\binom{n-3}{0}$	$\binom{n}{4}\binom{n-4}{0}$	—	—	$\binom{n}{n-1}\binom{n-(n-1)}{0}$	$\binom{n}{n}$
1	$\binom{n}{1}$	$\binom{n}{1}\binom{n-1}{1}$	$\binom{n}{2}\binom{n-2}{1}$	$\binom{n}{3}\binom{n-3}{1}$	$\binom{n}{4}\binom{n-4}{1}$	—	—	$\binom{n}{n-1}\binom{n-(n-1)}{1}$	
2	$\binom{n}{2}$	$\binom{n}{1}\binom{n-1}{2}$	$\binom{n}{2}\binom{n-2}{2}$	$\binom{n}{3}\binom{n-3}{2}$	$\binom{n}{4}\binom{n-4}{2}$	—	—		
3	$\binom{n}{3}$	$\binom{n}{1}\binom{n-1}{3}$	$\binom{n}{2}\binom{n-2}{3}$	$\binom{n}{3}\binom{n-3}{3}$	$\binom{n}{4}\binom{n-4}{3}$	—	—		
4	$\binom{n}{4}$	$\binom{n}{1}\binom{n-1}{4}$	$\binom{n}{2}\binom{n-2}{4}$	$\binom{n}{3}\binom{n-3}{4}$	$\binom{n}{4}\binom{n-4}{4}$	—	—		
—	—	—	—						
—	—	—							
n-1	$\binom{n}{n-1}$	$\binom{n}{1}\binom{n-1}{n-1}$							
n	$\binom{n}{n}$								
	S_0	S_1	S_2	S_3	S_4			S_{n-1}	S_n

($i = 0, 1, 2, 3, \dots, n$) represent the sum of the values of q_i^{th} ($i = 0, 1, 2, 3, \dots, n$) column respectively.

Using binomial expansion

$$S_0 = \binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \binom{n}{3} + \binom{n}{4} + \dots + \binom{n}{n-1} + \binom{n}{n} = (1+1)^n = 2^n$$

$$\begin{aligned} S_1 &= \binom{n}{1}\binom{n-1}{0} + \binom{n}{1}\binom{n-1}{1} + \binom{n}{2}\binom{n-1}{2} + \binom{n}{3}\binom{n-1}{3} + \binom{n}{4}\binom{n-1}{4} + \dots + \binom{n}{n-1}\binom{n-1}{n-1} \\ &= \binom{n}{1}[(\binom{n-1}{0} + \binom{n-1}{1} + \binom{n-1}{2} + \binom{n-1}{3} + \binom{n-1}{4} + \dots + \binom{n-1}{n-1})] \\ &= \binom{n}{1}(1+1)^{n-1} = \binom{n}{1}2^{n-1} \end{aligned}$$

$$\begin{aligned} S_2 &= \binom{n}{2}\binom{n-2}{0} + \binom{n}{2}\binom{n-2}{1} + \binom{n}{2}\binom{n-2}{2} + \binom{n}{2}\binom{n-2}{3} + \binom{n}{2}\binom{n-2}{4} + \dots + \binom{n}{2}\binom{n-2}{n-2} \\ &= \binom{n}{2}[(\binom{n-2}{0} + \binom{n-2}{1} + \binom{n-2}{2} + \binom{n-2}{3} + \binom{n-2}{4} + \dots + \binom{n-2}{n-2})] \\ &= \binom{n}{2}(1+1)^{n-2} = \binom{n}{2}2^{n-2} \end{aligned}$$

$$\begin{aligned} S_{n-1} &= \binom{n}{n-1}\binom{n-(n-1)}{0} + \binom{n}{n-1}\binom{n-(n-1)}{1} = \binom{n}{n-1}[\binom{1}{0} + \binom{1}{1}] \\ &= \binom{n}{n-1}(1+1)^1 = \binom{n}{n-1}2^1 \end{aligned}$$

$$S_n = \binom{n}{n} = \binom{n}{n}2^0.$$

Hence, the possible number of intuitionistic sets of a nonempty set with 'n' elements is given by

$$S_0 + S_1 + S_2 + S_3 + S_4 + \dots + S_{n-1} + S_n = 2^n + \binom{n}{1}2^{n-1} + \binom{n}{2}2^{n-2} + \dots + \binom{n}{n-1}2^1 + \binom{n}{n}2^0 = (1+2)^n = 3^n. \text{ Hence the proof. } \square$$

Example 3.2. Let $X = \{a, b\}$. Then the intuitionistic sets of X are $\langle X, \{a\}, \phi \rangle$, $\langle X, \{b\}, \phi \rangle$, $\langle X, \{a\}, \{b\} \rangle$, $\langle X, \{b\}, \{a\} \rangle$, $\langle X, \phi, \{a\} \rangle$, $\langle X, \phi, \{b\} \rangle$, $\langle X, \phi, \phi \rangle$, $\langle X, \phi, X \rangle$ and $\langle X, X, \phi \rangle$. Hence, the possible number of intuitionistic sets of a set with '2' elements is 3^2 .

Example 3.3. Let $X = \{a, b, c\}$. Then the possible intuitionistic sets of X are $\langle X, \{a, b\}, \phi \rangle$, $\langle X, \{b, c\}, \phi \rangle$, $\langle X, \{a\}, \phi \rangle$, $\langle X, \{b\}, \phi \rangle$, $\langle X, \{c\}, \phi \rangle$, $\langle X, \{a, c\}, \phi \rangle$, $\langle X, \{a\}, \{b\} \rangle$, $\langle X, \{a\}, \{c\} \rangle$, $\langle X, \{b\}, \{a\} \rangle$, $\langle X, \{b\}, \{c\} \rangle$, $\langle X, \{c\}, \{a\} \rangle$, $\langle X, \{c\}, \{b\} \rangle$, $\langle X, \phi, \{a\} \rangle$, $\langle X, \phi, \{b\} \rangle$, $\langle X, \phi, \{c\} \rangle$, $\langle X, \phi, \{a, b\} \rangle$, $\langle X, \phi, \{b, c\} \rangle$, $\langle X, \phi, \{a, c\} \rangle$, $\langle X, \{a, b\}, \{c\} \rangle$,

$\langle X, \{a, c\}, \{b\} \rangle, \langle X, \{b, c\}, \{a\} \rangle, \langle X, \{a\}, \{b, c\} \rangle, \langle X, \{b\}, \{a, c\} \rangle, \langle X, \{c\}, \{a, b\} \rangle, \langle X, \phi, \phi \rangle, \langle X, \phi, X \rangle$ and $\langle X, X, \phi \rangle$. Here, the number of intuitionistic sets is 27 which shows that the number of intuitionistic sets of a set with '3' elements is $3^3 = 27$.

Remark 3.4. Let X be a nonempty set with 'n' elements and (X, τ) be an intuitionistic topological space. Then the maximum number of intuitionistic open sets of an intuitionistic topological space is 3^n .

Remark 3.5. The general topological space becomes a subcategory of the intuitionistic topological space.

For, any topological space (X, τ_0) is obviously an ITS in the form $\tau = \{A' : A \in \tau_0\}$ whenever a subset A in X is identified with its counterpart $A' = \langle X, A, A^c \rangle$ or we can say $\tau = \{[A : A \in \tau_0]\}$.

4 Generators of an intuitionistic set

In this section, a new class of sets called intuitionistic generator and intuitionistic prime generator are defined using the operations union and complement between sets and intuitionistic sets respectively.

Definition 4.1. Let X be a nonempty set and $A = \langle X, A_1, A_2 \rangle$ be an intuitionistic set of X . Then the intuitionistic generator of A , denoted as $\mathcal{G}(A)$, is defined as the collection of intuitionistic sets of the form $\langle X, A_1, A_2 \rangle, \langle X, A_2, A_1 \rangle, \langle X, \phi, A_1 \cup A_2 \rangle$ and $\langle X, A_1 \cup A_2, \phi \rangle$.

Definition 4.2. Let X be a nonempty set and $A = \langle X, A_1, A_2 \rangle$ be an intuitionistic set of X . Then the intuitionistic prime generator of A , denoted as $\mathcal{G}_p(A)$, is defined as the collection of intuitionistic sets of the form $\langle X, A_1, \phi \rangle, \langle X, \phi, A_2 \rangle, \langle X, A_1, A_2 \rangle, \langle X, \phi, A_1 \cup A_2 \rangle$ and $\langle X, A_1 \cup A_2, \phi \rangle$.

Example 4.3. Let $X = \{a, b, c\}$ and $A = \langle X, \{a\}, \{b\} \rangle$. Then the intuitionistic generators of A i.e., $\mathcal{G}(A) = \{\langle X, \{a, b\}, \phi \rangle, \langle X, \phi, \{a, b\} \rangle, \langle X, \{a\}, \{b\} \rangle, \langle X, \{b\}, \{a\} \rangle\}$ and the intuitionistic prime generators of A i.e., $\mathcal{G}_p(A) = \{\langle X, \{a, b\}, \phi \rangle, \langle X, \phi, \{a, b\} \rangle, \langle X, \{a\}, \{b\} \rangle, \langle X, \{a\}, \phi \rangle, \langle X, \phi, \{b\} \rangle\}$.

Remark 4.4. If an intuitionistic set A is of the form $\langle X, \phi, A \rangle$ or $\langle X, A, \phi \rangle$ then $\mathcal{G}(A)$ and $\mathcal{G}_p(A)$ contain the same collection of nonempty intuitionistic sets.

Remark 4.5. For an intuitionistic set $A = \langle X, A_1, A_2 \rangle$, if $A_1 \cup A_2 = X$, then $\langle X, \phi, A_1 \cup A_2 \rangle$ and $\langle X, A_1 \cup A_2, \phi \rangle$ become ϕ and X respectively which imply $\mathcal{G}(A)$ and $\mathcal{G}_p(A)$ contain ϕ and X respectively.

Proposition 4.6. Let $A = \langle X, A_1, A_2 \rangle$ be an intuitionistic set of X such that $A_1 \cup A_2 = X$. Then both intuitionistic generators and intuitionistic prime generators have a topological structure. (i.e $\mathcal{G}(A)$ and $\mathcal{G}_p(A)$ are intuitionistic topologies.)

Proof. By Remark 4.5, ϕ and $\tilde{X} \in \mathcal{G}(A)$ and $\mathcal{G}_p(A)$. And since $A_1 \cup A_2 = X$ and $A_1 \cap A_2 = \phi$, the arbitrary union of intuitionistic sets and finite intersection of intuitionistic sets in $\mathcal{G}(A)$ and $\mathcal{G}_p(A)$ also belong to $\mathcal{G}(A)$ and $\mathcal{G}_p(A)$ respectively. $\square$

Proposition 4.7. *Let $A = \langle X, A_1, A_2 \rangle$ be an intuitionistic set of X such that $A_1 \cup A_2 = X$. Then $\mathcal{G}(A)$ itself is an intuitionistic fuzzy special σ - algebra.*

Proof. (i) Since $A_1 \cup A_2 = X$, $\langle X, A_1 \cup A_2, \phi \rangle = \tilde{X} \in \mathcal{G}(A)$, (ii) by Definition 4.1 of $\mathcal{G}(A)$, $A \in \mathcal{G}(A)$ implies $A^c \in \mathcal{G}(A)$ and (iii) union of any two intuitionistic sets in $\mathcal{G}(A)$ belongs to $\mathcal{G}(A)$. Hence $\mathcal{G}(A)$ itself is an intuitionistic fuzzy special σ - algebra. $\square$

Remark 4.8. *If the condition $A_1 \cup A_2 = X$ does not hold then the intuitionistic generator $\mathcal{G}(A)$ does not form an intuitionistic fuzzy special σ - algebra because $\tilde{X} \notin \mathcal{G}(A)$.*

Example 4.9. *Let $X = \{a, b, c\}$ and $A = \langle X, \{a\}, \{c\} \rangle$. Then the intuitionistic generators of A i.e., $\mathcal{G}(A) = \{\langle X, \{a, c\}, \phi \rangle, \langle X, \phi, \{a, c\} \rangle, \langle X, \{a\}, \{c\} \rangle, \langle X, \{c\}, \{a\} \rangle\}$ does not form an intuitionistic fuzzy special σ - algebra because $\tilde{X} \notin \mathcal{G}(A)$.*

Remark 4.10. *The intuitionistic prime generator $\mathcal{G}_{(p)}(A)$ does not form an intuitionistic fuzzy special σ - algebra under any circumstances, because condition (ii) in Definition 3.1 of [3] never holds good in $\mathcal{G}_{(p)}(A)$.*

5 Intuitionistic generator topology

In this section, intuitionistic generator topological space and intuitionistic prime generator topological space generated by any intuitionistic set are formed and their characterizations are studied.

Definition 5.1. *Let X be a nonempty set and A be any intuitionistic set in X . The collection $\mathcal{G}(A)$ along with ϕ and $\tilde{X}$ forms a topology and it is called intuitionistic generator topology generated by A and is denoted by $(X, \tau_{\mathcal{G}})$.*

Thus corresponding to each intuitionistic set A of X , there exists an intuitionistic generator topology $\tau_{\mathcal{G}}$ generated by A . The intuitionistic sets in $(X, \tau_{\mathcal{G}})$ are the IG open sets of X and the collection of IG open sets generated by an intuitionistic set is denoted as $IGO(X)$. The complement of IG open sets are called IG closed sets and is denoted by $IGC(X)$.

Remark 5.2. *For a two element set X , the possible number of intuitionistic topologies generated by any intuitionistic set is only four.*

Example 5.3. *Let X be a nonempty set with two elements a, b . Then the possible intuitionistic topologies generated by the different intuitionistic sets of X are given by*

1. $\tau_{\mathcal{G}} = \{\phi, \tilde{X}\}$ 2. $\tau_{\mathcal{G}} = \{\phi, \tilde{X}, \langle X, \{a\}, \{b\} \rangle, \langle X, \{b\}, \{a\} \rangle\}$
3. $\tau_{\mathcal{G}} = \{\phi, \tilde{X}, \langle X, \{a\}, \phi \rangle, \langle X, \phi, \{a\} \rangle\}$ 4. $\tau_{\mathcal{G}} = \{\phi, \tilde{X}, \langle X, \{b\}, \phi \rangle, \langle X, \phi, \{b\} \rangle\}$.

Proposition 5.4. *Let X be a nonempty set and A be any intuitionistic set of X . In an intuitionistic topological space (X, τ_G) , $IGO(X) = IGC(X)$.*

Proof. Since intuitionistic set and its complement always occur in pairs in $\mathcal{G}(A)$, the result holds. $\square$

Definition 5.5. *Let X be a nonempty set and A be any intuitionistic set of X . The collection $\mathcal{G}_p(A)$ along with ϕ and $\tilde{X}$ forms a topology and it is called intuitionistic prime generator topology generated by A and is denoted by (X, τ_{PG}) .*

The intuitionistic sets in (X, τ_{PG}) are the IPG open sets of X and the collection of IPG open sets generated by an intuitionistic set is denoted as $IG_pO(X)$. The complement of IPG open sets is called IPG closed sets and is denoted by $IG_pC(X)$.

Proposition 5.6. *Let X be a nonempty set and A be any intuitionistic set of X . Then the intuitionistic topological space (X, τ_G) is always intuitionistic C_5 -disconnected.*

Proof. Let A be an intuitionistic open set of an intuitionistic topological space (X, τ_G) . By the definition of $\mathcal{G}(A)$, A and A^c always belong to $\mathcal{G}(A)$. So, (X, τ_G) contains intuitionistic set which is both intuitionistic open and closed. Hence it is intuitionistic C_5 -disconnected. $\square$

Proposition 5.7. *Let $A = \langle X, A_1, A_2 \rangle$ be an intuitionistic set of X . Then $IG_pO(X)$ is intuitionistic C_5 -disconnected only if $A_1 \cup A_2 \neq X$.*

Proof. If $A_1 \cup A_2 = X$, then by Definition 5.5, $IG_pO(X) = \{\phi, \tilde{X}, \langle X, A_1, \phi \rangle, \langle X, \phi, A_2 \rangle, \langle X, A_1, A_2 \rangle\}$ which does not contain proper intuitionistic set (i.e., ϕ and $\tilde{X}$) both intuitionistic open and closed. So, $IG_pO(X)$ is intuitionistic C_5 -connected.

On the other hand, if $A_1 \cup A_2 \neq X$, then $IG_pO(X)$ contains $\langle X, A_1 \cup A_2, \phi \rangle$, $\langle X, \phi, A_1 \cup A_2 \rangle$ which is both intuitionistic open and closed. Hence it is intuitionistic C_5 -disconnected. $\square$

Proposition 5.8. *Let $A = \langle X, A_1, A_2 \rangle$ be an intuitionistic set of X such that $A_1 \cup A_2 = X$. Then the intuitionistic topological space (X, τ_G) is intuitionistic disconnected.*

Proof. Since $\mathcal{G}(A)$ contains both A and A^c , $A \cup A^c = \langle X, A_1 \cup A_2, A_1 \cap A_2 \rangle = \langle X, X, \phi \rangle = \tilde{X}$ and $A \cap A^c = \langle X, A_1 \cap A_2, A_1 \cup A_2 \rangle = \langle X, \phi, X \rangle = \phi$. Hence (X, τ_G) is intuitionistic disconnected. $\square$

Definition 5.9. *Let (X, τ_G) be an intuitionistic generator topological space generated by any intuitionistic set. Then*

$$IGcl(B) = \bigcap \{K : K \in IGC(X) \text{ and } B \subseteq K\}$$

$$IGint(A) = \bigcup \{K : K \in IGO(X) \text{ and } K \subseteq A\}$$

Definition 5.10. *Let (X, τ_{PG}) be an intuitionistic prime generator topological space generated by any intuitionistic set. Then*

$$IG_pcl(B) = \bigcap \{K : K \in IG_pC(X) \text{ and } B \subseteq K\}$$

$$IG_pint(B) = \bigcup \{K : K \in IG_pO(X) \text{ and } K \subseteq B\}$$

Definition 5.11. An intuitionistic topological space (X, τ_G) is said to be *IG-extremally disconnected* if $IGcl(B) \in IGO(X)$ for every $B \in IGO(X)$.

Definition 5.12. An intuitionistic topological space (X, τ_{GP}) is said to be *IG_p-extremally disconnected* if $IGpcl(B) \in IGPO(X)$ for every $B \in IGPO(X)$.

Proposition 5.13. An intuitionistic topological space (X, τ_G) is always *IG-extremally disconnected*.

Proof. Since $IGcl(B)$ is *IG*-closed for every $B \in IGO(X)$ and by Proposition 5.4 $B, B^c \in IGO(X)$. Hence $IGcl(B) \in IGO(X)$. $\square$

Example 5.14. Let $X = \{a, b, c\}$ and $A = \langle X, \{a\}, \{b\} \rangle$. Then the intuitionistic generator topological space generated by A is $\{\phi, X, \langle X, \{a, b\}, \phi \rangle, \langle X, \phi, \{a, b\} \rangle, \langle X, \{a\}, \{b\} \rangle, \langle X, \{b\}, \{a\} \rangle\}$ in which $IGcl(B) \in IGO(X)$ for every $B \in IGO(X)$. Hence (X, τ_G) is *IG-extremally disconnected*.

Remark 5.15. (X, τ_{GP}) is not *IG_p-extremally disconnected*.

Example 5.16. Let $\{A\} = \{\langle X, \{a\}, \{b\} \rangle\}$ where $X = \{a, b, c\}$, then $IGPO(X) = \{\phi, X, \langle X, \{a, b\}, \phi \rangle, \langle X, \phi, \{a, b\} \rangle, \langle X, \{a\}, \{b\} \rangle, \langle X, \{a\}, \phi \rangle, \langle X, \phi, \{b\} \rangle\}$. Here $IGpcl(\langle X, \phi, \{b\} \rangle) = \langle X, \{b\}, \phi \rangle$ which does not belong to $IGPO(X)$. Hence (X, τ_{GP}) is not *IG_p-extremally disconnected*.

Proposition 5.17. Let (X, τ_G) be an intuitionistic generator topological space generated by an intuitionistic set, then it is always an intuitionistic R_0 -space.

Proof. Since every intuitionistic set of $IGO(X)$ is intuitionistic closed, every intuitionistic open set contains the intuitionistic closure of each of its intuitionistic points. Hence, (X, τ_G) is always an intuitionistic R_0 -space. $\square$

Remark 5.18. The intuitionistic topology (X, τ_{GP}) need not be an intuitionistic R_0 -space.

Example 5.19. Let $X = \{a, b, c\}$. Then the intuitionistic prime generator topology generated by $A = \langle X, \{a\}, \{b\} \rangle$ is $\{\phi, X, \langle X, \{a, b\}, \phi \rangle, \langle X, \phi, \{a, b\} \rangle, \langle X, \{a\}, \{b\} \rangle, \langle X, \{a\}, \phi \rangle, \langle X, \phi, \{b\} \rangle\}$. In this the intuitionistic set $\langle X, \{a\}, \{b\} \rangle$ does not contain the intuitionistic closure of its intuitionistic point a . Hence it is not an intuitionistic R_0 -space.

Definition 5.20. Let X be a nonempty set. Then the intuitionistic generator topological space (X, τ_G) generated by an intuitionistic set is said to be

- (i) *IG-semi open* if $B \subseteq IGcl(IGint(B))$.
- (ii) *IG-preopen* if $B \subseteq IGint(IGcl(B))$.
- (iii) *IG-regular open* if $B = IGint(IGcl(B))$.

The complement of intuitionistic *IG-semiopen* (resp. *IG-preopen*, *IG-regular open*) sets is called *IG-semiclosed* (resp. *IG-preclosed*, *IG-regular closed*). The family of *IG-semiopen* (resp. *IG-preopen*, *IG-regular open*) sets of an intuitionistic topological space (X, τ) is denoted by $IGSO(X)$ (resp. $IGPO(X)$, $IGRO(X)$).

Proposition 5.21. *Let $(X, \tau_{\mathcal{G}})$ be an intuitionistic generator topological space generated by an intuitionistic set. Then every IG open set is IG regular open.*

Proof. Since $IGcl(B)$ is IG -closed for every $B \in IGO(X)$ and by Proposition 5.4, $B = IGint(IGcl(B))$. Hence the result follows. $\square$

Proposition 5.22. *For an intuitionistic generator topological space $(X, \tau_{\mathcal{G}})$ generated by an intuitionistic set, every intuitionistic IG -regular open set of X is intuitionistic IG -closed.*

Proof. Let A be an IG -regular open set of $(X, \tau_{\mathcal{G}})$. Since $(X, \tau_{\mathcal{G}})$ is IG -extremally disconnected, $A = IGint(IGcl(A))$ and so $IGcl(A) \in \tau_{\mathcal{G}}$. Thus A is IG -open and hence A is IG -closed. $\square$

Remark 5.23. *In an intuitionistic generator topological space $(X, \tau_{\mathcal{G}})$ generated by an intuitionistic set, every intuitionistic IG -regular closed set of X is intuitionistic IG -open.*

Proposition 5.24. *Let $(X, \tau_{\mathcal{G}})$ be an intuitionistic generator topological space generated by an intuitionistic set. Then IG -regular open sets of X coincides with IG -regular closed sets.*

Proof. Proof follows from Proposition 5.21 and Theorem 5.22. $\square$

Proposition 5.25. *In an intuitionistic generator topological space $(X, \tau_{\mathcal{G}})$ generated by an intuitionistic set, every intuitionistic IG -regular closed set of X is intuitionistic IG -preopen.*

Proof. Since every IG -regular open set is IG -preopen and from Theorem 5.24, the result follows. $\square$

Theorem 5.26. *Let $(X, \tau_{\mathcal{G}})$ be an intuitionistic generator topological space generated by an intuitionistic set. If A and B are IG -regular closed, then $A \cap B$ is IG -regular closed.*

Proof. Let A and B be IG -regular closed sets of X . Since A and B are IG -closed, $IGint(A)$ and $IGint(B)$ are IG -closed as $(X, \tau_{\mathcal{G}})$ is IG -extremally disconnected. This implies that $A \cap B = IGcl(IGint(A)) \cap IGcl(IGint(B)) = IGint(A) \cap IGint(B) = IGint(A \cap B) \subset IGcl(IGint(A \cap B))$. On the other hand, $IGcl(IGint(A \cap B)) = IGcl(IGint(A)) \cap IGcl(IGint(B)) \subset IGcl(IGint(A)) \cap IGcl(IGint(B)) = A \cap B$. Thus $A \cap B$ is IG -regular closed. $\square$

Definition 5.27. *If there exists an IG -regular open set B in X such that $\phi \neq B \neq \tilde{X}$, then X is called IG -super disconnected. X is called IG -super connected, if X is not IG -super disconnected.*

Proposition 5.28. *If the intuitionistic generator topological space $(X, \tau_{\mathcal{G}})$ is not indiscrete then it is IG super disconnected.*

Proof. Let B be an intuitionistic open set of $(X, \tau_{\mathcal{G}})$. Since B and B^c belongs to $\mathcal{G}(A)$, there exists an IG -regular open set B such that $\phi \neq B \neq \tilde{X}$. Hence $(X, \tau_{\mathcal{G}})$ is IG -super disconnected. $\square$

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