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TWO STEP ESTIMATION - A NEW T-X MODEL

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Abstract. In this paper, a new T-X probability model is considered; specifically, we have taken Pareto model for T and Rayleigh distribution for X and named the resulting model as a T-X model named as Pareto-Rayleigh distribution. The ML method of estimation of the model doesn't possess any explicit equations and consists of two parameters, we are using a new approach called two step estimation process. The efficiency and preference of methods are established on basis of counts of generalized variances.

1. Introduction

Let $F(x)$ be the cumulative distribution function (CDF) of any random variable X and $r(t)$ be the probability density function (PDF)of a random variable, T, defined on $[0, \infty]$. The CDF of the T-X family of distributions defined by Alzaatreh, *et al.* (2012) is given by

$$G(x) = \int_0^{-\log[1-F(x)]} r(t)dt \tag{1.1}$$

Alzaatreh, *et al.* (2012) named this family of distributions as the Transformed-Transformer family (or T-X family). If a random variable T follows the Pareto distribution type IV with parameter α then

$$r(t, \alpha, \sigma) = \frac{\alpha}{\sigma}\left[1 + \frac{t}{\sigma}\right]^{-(\alpha+1)}$$

$$= \alpha\,[1 + t]^{-(\alpha+1)}\ ; \quad t > 0, \alpha > 1, \sigma = 1 \tag{1.2}$$

If a random variable X follows the Rayleigh distribution with parameter σ then

$$F(x) = 1 - e^{-x^2/2\sigma^2}; \qquad x > 0, \sigma > 0 \tag{1.3}$$

Using (1.1), (1.2) and (1.3), we obtain a new T-X family of distribution called Pareto-Rayleigh distribution (P-R distribution) and its CDF is given by

$$G(x) = 1 - \left[1 + \frac{x^2}{2\sigma^2}\right]^{-\alpha} \ ; \quad x > 0, \alpha > 1, \sigma > 0 \tag{1.4}$$

The probability density function (pdf) corresponding to (1.4) is

$$g(x) = \frac{\alpha}{\sigma^2} x \left[1 + \frac{x^2}{2\sigma^2}\right]^{-\alpha-1} \ ; \quad x > 0, \alpha > 1, \sigma > 0 \tag{1.5}$$

where α is shape parameter and σ is scale parameter.

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2. Two-Step Estimation

Many lifetime data used for statistical analysis follow a particular statistical distribution. Knowledge of the appropriate distribution that any phenomenon follows, greatly improves the sensitivity, power and efficiency of the statistical tests associated with it. Several distributions exist for modelling these lifetime data. However, some of these lifetime data do not follow these existing distributions or are inappropriately described by them. As a result, plenty of distributions have been developed and studied by researchers. The parametric estimation may be estimated by Classical MMLE Method discussed by several authors. Tikku (1967), Pearson and Rootzen(1977), Rosiah *et. al.* (1993) Kantam and sriram (2003), Subba rao *et. al.* (2010).

In the multi-parameter case if all the parameters are unknown and when classical ML method of estimation doesn't give explicit estimators, the parameters are successively estimated stepwise by different simpler methods without sacrificing the joint efficiencies. Such methods are called multi-step methods of estimation.

In particular, for the two-parameter case it is called two-step estimation. Such estimation is done by Engeman and Keefe (1985) for the Weibull distribution, Srinivasa Rao and Kantam (2012) for log-logistic distribution, Ravi Kumar and Kantam (2016) for Burr Type X distribution and the references there in.

In this method, we estimate the two parameters α and σ by two different methods in succession. By inverting the distribution function of P-R Model is

$$\begin{aligned} Let\ \ G(x)\ &= p_i \\ 1-\left[1+\tfrac{x^2}{2}\right]^{-\alpha} &= p_i \\ \left[1+\tfrac{x^2}{2}\right]^{-\alpha} &= 1-p_i \end{aligned}$$

$$\alpha = \frac{-\log\left(1-p_i\right)}{\log\left(1+\frac{x_i^2}{2}\right)}, \quad i = 1, 2, ...n \tag{2.1}$$

where $p_i = \frac{i}{n+1}$and x_i is i^{th} ordered statistic. That is, for a given ordered sample of size 'n', we get 'n' estimates of α corresponding to each ordered observation. In order to use collectively all these sample observations, we propose the A.M, Median, G.M, H.M of the n estimates of α. If a sample is generated with a known α we get thus four different estimates of α in the 1^{st} step of estimation.

The second parameter α can be estimated by any method of estimation. It is known as 2^{nd} step of estimation. In this paper, the α is taken that derived by Subba Rao *et. al.* (2015) using MMLE I and MMLE II of the same probability model.

$$\hat{\sigma} = \frac{\sum \delta_i x_i}{\frac{n}{\alpha+1} - \sum \gamma_i} \tag{2.2}$$

The MMLE of σ can be obtained if δi and γi are known and two methods are proposed to get δi and γi called as MMLE I and MMLE II. Also, equation (2.2) consists of α which can be obtained from Step 1. Jointly the two unknown parameters (α, σ) are defined to be estimated by two-step estimation. Thus, for a given sample of size 'n' using two step estimation we get 8 pairs of estimators (i.e.) 4 types of Averages in Step 1 and 2 Methods of estimation in Step 2. The performances of such pairs of estimators are studied by simulation with the help of the notion of generalized variances trace and determinant of the empirical covariance matrix of the estimators. We have considered for various α values namely α = 2, 3, 4 and the sample size n = 5(1.5)25. The computed estimates, their means, variances, MSEs and Co-variances followed by generalized variances are given in the following tables 2.1 to 2.6.

3. Comparison and Conclusion

The marked positions denoted with (*) of the tables 2.4 to 2.6 indicate the preference of the corresponding pair of two step estimation listed in the columns I and II. The counts of generalized variances for two parameters are given in table 3.1

Table 3.1
The counts of generalized variances for two parameters

α	Step I of Method Estimation α	Step II of Method of Estimation σ	Counts of Generalized Variances	
			I	II
2	HM-Method	MMLE – II	5	5
	HM-Method	MMLE – I	-	2
	Median-Method	MMLE – I	-	1
	Median - Method	MMLE – II	-	1
3	HM - Method	MMLE – II	5	4
	Median - Method	MMLE – I	-	1
	Median - Method	MMLE – II	-	1
4	HM - Method	MMLE – II	5	4
	Median - Method	MMLE – I	-	1
	Median - Method	MMLE – II	-	1

By observing the frequencies of minimum positions occupied by the two types of generalized variances for various pairs of methods of estimation in the two steps successively applied, we noticed that type I generalized variances has not given minimum value at many combinations. We therefore rely on type II generalized variance only. It has occupied minimum place at maximum count of 5 places for the pair of methods namely, HM – Method in step – I and MMLE – II Method in Step – II, leading to a two-step estimation is preferable.

Table 2.1
Two Step Estimation - Mean, Variance, Co-variance and MSE of α and σ at $\alpha = 2$

Sample Size n	Averages	Mean(α)	Var(α)	Mean (σ)		Var(σ)		Cov(α, σ)		MSE(α)
				MMLE I	MMLE II	MMLE I	MMLE II	MMLE I	MMLE II	
5	A.M.	5.58587	1794.69686	1.28145	1.31171	0.39933	0.48594	8.45859	10.20363	1807.5553
	Median	2.52515	4.05871	1.06331	1.07132	0.05969	0.05252	0.13622	0.15689	4.3345
	G.M.	2.70694	4.53839	1.10148	1.11226	0.05789	0.05366	0.13370	0.16571	5.0382
	H.M.	2.25455	2.16932	1.00772	1.01403	0.01731	0.01310	-0.03727	-0.02113	2.2341
10	A.M.	3.67861	383.40953	1.25665	1.26758	0.46789	0.46806	5.82804	5.82272	386.2273
	Median	2.22189	0.96580	1.03351	1.04294	0.01597	0.01385	0.04319	0.04154	1.0150
	G.M.	2.35687	1.19622	1.06943	1.07940	0.02201	0.02063	0.07234	0.07216	1.3236
	H.M.	2.10355	0.72031	1.00140	1.01097	0.00781	0.00680	0.00876	0.00893	0.7310
15	A.M.	3.42288	255.79788	1.23979	1.24285	0.66336	0.56600	8.95752	7.99935	257.8225
	Median	2.12907	0.50522	1.02220	1.03143	0.00854	0.00768	0.02478	0.02313	0.5219
	G.M.	2.24366	0.61856	1.05597	1.06522	0.01361	0.01296	0.04478	0.04333	0.6779
	H.M.	2.058141	0.42321	1.00124	1.01067	0.00539	0.00500	0.01175	0.01090	0.4266
20	A.M.	3.06070	143.93128	1.21171	1.21241	0.60713	0.48264	6.42277	5.39987	145.0564
	Median	2.10347	0.35573	1.01634	1.02493	0.00534	0.00495	0.01788	0.01649	0.3664
	G.M.	2.20038	0.44233	1.04551	1.05395	0.00969	0.00935	0.03495	0.03353	0.4825
	H.M.	2.04977	0.32108	0.99984	1.00857	0.00433	0.00416	0.01300	0.01203	0.3236
25	A.M.	2.80600	39.33183	1.19660	1.19669	0.60400	0.47113	3.89578	3.26618	39.9815
	Median	2.08588	0.27887	1.01294	1.02077	0.00362	0.00343	0.01340	0.01234	0.2862
	G.M.	2.17422	0.34960	1.04022	1.04786	0.00788	0.00767	0.02888	0.02769	0.3800
	H.M.	2.04352	0.26545	0.99970	1.00763	0.00372	0.00363	0.01262	0.01177	0.2673

Table 2.2

Two Step Estimation - Mean, Variance, Co-variance and MSE of α and σ at $\alpha = 3$

Sample Size n	Averages	Mean(α)	Var(α)	Mean (σ)		Var(σ)		Cov(α,σ)		MSE(α)
				MMLE I	MMLE II	MMLE I	MMLE II	MMLE I	MMLE II	
5	A.M.	8.40134	7252.25123	1.19535	1.19252	0.17545	0.18214	6.61007	6.92456	7293.2284
	Median	3.76772	9.76390	1.02640	1.02038	0.03061	0.02847	0.17249	0.17588	12.8887
	G.M.	4.02294	10.27193	1.06323	1.05792	0.03301	0.03221	0.16100	0.17077	14.3642
	H.M.	3.34237	3.95688	0.98076	0.97487	0.00964	0.00875	-0.02345	-0.01797	5.7588
10	A.M.	5.51791	862.67144	1.17908	1.17605	0.15200	0.14740	3.59831	3.52693	875.0472
	Median	3.33284	2.17305	1.01823	1.01682	0.01201	0.01098	0.06057	0.05682	3.9495
	G.M.	3.53530	2.69150	1.05055	1.04903	0.01753	0.01672	0.09707	0.09379	5.0486
	H.M.	3.15532	1.62069	0.98947	0.98846	0.00661	0.00617	0.02149	0.01986	2.9555
15	A.M.	5.13432	575.54523	1.16270	1.16090	0.13375	0.12696	4.02008	3.87399	585.3692
	Median	3.19360	1.13674	1.01322	1.01389	0.00731	0.00682	0.03713	0.03477	2.5614
	G.M.	3.36549	1.39176	1.04376	1.04416	0.01188	0.01141	0.06313	0.06063	3.2563
	H.M.	3.08721	0.95222	0.99422	0.99513	0.00531	0.00507	0.02230	0.02076	2.1342
20	A.M.	4.59105	323.84538	1.14313	1.14223	0.11099	0.10491	2.57371	2.46906	330.5589
	Median	3.15521	0.80040	1.01039	1.01187	0.00497	0.00472	0.02757	0.02581	2.1349
	G.M.	3.30056	0.99524	1.03663	1.03785	0.00887	0.00859	0.05031	0.04832	2.6867
	H.M.	3.07465	0.72243	0.99518	0.99683	0.00453	0.00438	0.02273	0.02136	1.8773
25	A.M.	4.20900	88.49663	1.13460	1.13415	0.10693	0.10103	1.69007	1.62597	93.3763
	Median	3.12882	0.62745	1.00851	1.01031	0.00367	0.00352	0.02170	0.02037	1.9017
	G.M.	3.26133	0.78661	1.03309	1.03465	0.00743	0.00723	0.04243	0.04084	2.3776
	H.M.	3.06528	0.59727	0.99618	0.99810	0.00401	0.00390	0.02167	0.02053	1.7321

Table 2.3

Two Step Estimation - Mean, Variance, Co-variance and MSE of α and σ at $\alpha = 4$

Sample Size n	Averages	Mean(α)	Var(α)	Mean (σ)		Var(σ)		Cov(α,σ)		MSE(α)
				MMLE I	MMLE II	MMLE I	MMLE II	MMLE I	MMLE II	
5	A.M.	11.17174	7178.78746	1.16609	1.15253	0.14240	0.14013	7.49379	7.42319	7262.9083
	Median	5.05031	16.23483	1.01101	0.99908	0.02833	0.02688	0.20894	0.20041	25.5392
	G.M.	5.41389	18.15355	1.04633	1.03428	0.03116	0.03063	0.21304	0.20979	29.8082
	H.M.	4.50909	8.67728	0.96586	0.95487	0.00773	0.00761	-0.01627	-0.01695	14.9728
10	A.M.	7.35722	1533.63812	1.15171	1.14296	0.10552	0.10114	3.59242	3.49243	1562.3379
	Median	4.44378	3.86319	1.00991	1.00368	0.01082	0.01010	0.08090	0.07551	9.8353
	G.M.	4.71374	4.78488	1.04055	1.03395	0.01608	0.01540	0.12562	0.12001	12.1492
	H.M.	4.20710	2.88123	0.98261	0.97688	0.00645	0.00614	0.03553	0.03248	7.7525
15	A.M.	6.84576	1023.19151	1.13911	1.13352	0.08761	0.08372	3.80438	3.68908	1046.6729
	Median	4.25813	2.02088	1.00797	1.00467	0.00691	0.00654	0.05039	0.04745	7.1200
	G.M.	4.48731	2.47425	1.03688	1.03322	0.01121	0.01080	0.08256	0.07924	8.6610
	H.M.	4.11628	1.69284	0.98987	0.98683	0.00539	0.00518	0.03342	0.03131	6.1715
20	A.M.	6.12140	575.72511	1.12277	1.11897	0.07170	0.06865	2.41874	2.34649	592.7110
	Median	4.20695	1.42293	1.00671	1.00482	0.00488	0.00467	0.03774	0.03571	6.2936
	G.M.	4.40075	1.76931	1.03144	1.02927	0.00852	0.00825	0.06605	0.06364	7.5329
	H.M.	4.09954	1.28431	0.99212	0.99041	0.00467	0.00453	0.03273	0.03106	5.6924
25	A.M.	5.61200	157.32734	1.11604	1.11324	0.06842	0.06565	1.62624	1.58280	170.3739
	Median	4.17176	1.11547	1.00564	1.00454	0.00371	0.00358	0.03015	0.02864	5.8320
	G.M.	4.34844	1.39842	1.02881	1.02746	0.00720	0.00700	0.05603	0.05417	6.9136
	H.M.	4.08704	1.06182	0.99378	0.99280	0.00415	0.00404	0.03083	0.02948	5.4176

Table 2.4
Generalized Variances of Two-Step estimation at $\alpha = 2$

n	Step 1 of method of estimation α	Step 2 of method of estimation σ	Var($\boldsymbol{\alpha}$)	Var(σ)	Cov(α,σ)	Generalized Variances	
						I	II
5	AM – Method	MMLE - I	1794.69686	0.39933	8.45859	1795.0962	645.1346
	Median – Method	MMLE - I	4.05871	0.05969	0.13622	4.1184	0.2237
	GM – Method	MMLE - I	4.53839	0.05789	0.13370	4.5963	0.2448
	HM – Method	MMLE - I	2.16932	0.01731	-0.03727	2.1866	0.0362
	AM – Method	MMLE - II	1794.69686	0.48594	10.20363	1795.1828	768.0061
	Median – Method	MMLE - II	4.05871	0.05252	0.15689	4.1112	0.1885
	GM – Method	MMLE - II	4.53839	0.05366	0.16571	4.5920	0.2161
	HM – Method	MMLE - II	2.16932	0.01310	-0.02113	2.1824*	0.0280*
10	AM – Method	MMLE - I	383.40953	0.46789	5.82804	383.8774	145.4278
	Median – Method	MMLE - I	0.96580	0.01597	0.04319	0.9818	0.0136
	GM – Method	MMLE - I	1.19622	0.02201	0.07234	1.2182	0.0211
	HM – Method	MMLE - I	0.72031	0.00781	0.00876	0.7281	0.0055
	AM – Method	MMLE - II	383.40953	0.46806	5.82272	383.8776	145.5527
	Median – Method	MMLE - II	0.96580	0.01385	0.04154	0.9796	0.0117
	GM – Method	MMLE - II	1.19622	0.02063	0.07216	1.2168	0.0195
	HM – Method	MMLE - II	0.72031	0.00680	0.00893	0.7271*	0.0048*
15	AM – Method	MMLE - I	255.79788	0.66336	8.95752	256.4612	89.4495
	Median – Method	MMLE - I	0.50522	0.00854	0.02478	0.5138	0.0037
	GM – Method	MMLE - I	0.61856	0.01361	0.04478	0.6322	0.0064
	HM – Method	MMLE - I	0.42321	0.00539	0.01175	0.4286	0.0021
	AM – Method	MMLE - II	255.79788	0.56600	7.99935	256.3639	80.7914
	Median – Method	MMLE - II	0.50522	0.00768	0.02313	0.5129	0.0033
	GM – Method	MMLE - II	0.61856	0.01296	0.04333	0.6315	0.0061
	HM – Method	MMLE - II	0.42321	0.00500	0.01090	0.4282*	0.0020*
20	AM – Method	MMLE - I	143.9313	0.60713	6.42277	144.5384	46.1326
	Median – Method	MMLE - I	0.3557	0.00534	0.01788	0.3611	0.0016
	GM – Method	MMLE - I	0.4423	0.00969	0.03495	0.4520	0.0031
	HM – Method	MMLE - I	0.3211	0.00433	0.01300	0.3254	0.0012*
	AM – Method	MMLE - II	143.9313	0.48264	5.39987	144.4139	40.3088
	Median – Method	MMLE - II	0.3557	0.00495	0.01649	0.3607	0.0015
	GM – Method	MMLE - II	0.4423	0.00935	0.03353	0.4517	0.0030
	HM – Method	MMLE - II	0.3211	0.00416	0.01203	0.3252*	0.0012*
25	AM – Method	MMLE - I	39.3318	0.60400	3.89578	39.9358	8.5793
	Median – Method	MMLE - I	0.2789	0.00362	0.01340	0.2825	0.0008*
	GM - Method	MMLE - I	0.3496	0.00788	0.02888	0.3575	0.0019
	HM - Method	MMLE - I	0.2655	0.00372	0.01262	0.2692	0.0008*
	AM - Method	MMLE - II	39.3318	0.47113	3.26618	39.8030	7.8626
	Median - Method	MMLE - II	0.2789	0.00343	0.01234	0.2823	0.0008*
	GM - Method	MMLE - II	0.3496	0.00767	0.02769	0.3573	0.0019
	HM - Method	MMLE - II	0.2655	0.00363	0.01177	0.2691*	0.0008*

Table 2.5
Generalized Variances of Two-Step estimation at $\alpha = 3$

n	Step 1 of method of estimation α	Step 2 of method of estimation σ	Var($\boldsymbol{\alpha}$)	Var(σ)	Cov(α, σ)	Generalized Variances	
						I	II
5	AM - Method	MMLE - I	7252.25123	0.17545	6.61007	7252.4267	1228.7426
	Median - Method	MMLE - I	9.76390	0.03061	0.17249	9.7945	0.2692
	GM - Method	MMLE - I	10.27193	0.03301	0.16100	10.3049	0.3132
	HM - Method	MMLE - I	3.95688	0.00964	-0.02345	3.9665	0.0376
	AM - Method	MMLE - II	7252.25123	0.18214	6.92456	7252.4334	1272.9725
	Median - Method	MMLE - II	9.76390	0.02847	0.17588	9.7924	0.2471
	GM - Method	MMLE - II	10.27193	0.03221	0.17077	10.3041	0.3017
	HM - Method	MMLE - II	3.95688	0.00875	-0.01797	3.9656*	0.0343*
10	AM - Method	MMLE - I	862.67144	0.15200	3.59831	862.8234	118.1758
	Median - Method	MMLE - I	2.17305	0.01201	0.06057	2.1851	0.0224
	GM - Method	MMLE - I	2.69150	0.01753	0.09707	2.7090	0.0378
	HM - Method	MMLE - I	1.62069	0.00661	0.02149	1.6273	0.0102
	AM - Method	MMLE - II	862.67144	0.14740	3.52693	862.8188	114.7219
	Median - Method	MMLE - II	2.17305	0.01098	0.05682	2.1840	0.0206
	GM - Method	MMLE - II	2.69150	0.01672	0.09379	2.7082	0.0362
	HM - Method	MMLE - II	1.62069	0.00617	0.01986	1.6269*	0.0096*
15	AM - Method	MMLE - I	575.54523	0.13375	4.02008	575.6790	60.8155
	Median - Method	MMLE - I	1.13674	0.00731	0.03713	1.1441	0.0069
	GM - Method	MMLE - I	1.39176	0.01188	0.06313	1.4036	0.0125
	HM - Method	MMLE - I	0.95222	0.00531	0.02230	0.9575	0.0046
	AM - Method	MMLE - II	575.54523	0.12696	3.87399	575.6722	58.0644
	Median - Method	MMLE - II	1.13674	0.00682	0.03477	1.1436	0.0065
	GM - Method	MMLE - II	1.39176	0.01141	0.06063	1.4032	0.0122
	HM - Method	MMLE - II	0.95222	0.00507	0.02076	0.9573*	0.0044*
20	AM - Method	MMLE - I	323.8454	0.11099	2.57371	323.9564	29.3198
	Median - Method	MMLE - I	0.8004	0.00497	0.02757	0.8054	0.0032
	GM - Method	MMLE - I	0.9952	0.00887	0.05031	1.0041	0.0063
	HM - Method	MMLE - I	0.7224	0.00453	0.02273	0.7270	0.0028
	AM - Method	MMLE - II	323.8454	0.10491	2.46906	323.9503	27.8783
	Median - Method	MMLE - II	0.8004	0.00472	0.02581	0.8051	0.0031
	GM - Method	MMLE - II	0.9952	0.00859	0.04832	1.0038	0.0062
	HM - Method	MMLE - II	0.7224	0.00438	0.02136	0.7268*	0.0027*
25	AM – Method	MMLE - I	88.4966	0.10693	1.69007	88.6036	6.6070
	Median – Method	MMLE - I	0.6275	0.00367	0.02170	0.6311	0.0018*
	GM – Method	MMLE - I	0.7866	0.00743	0.04243	0.7940	0.0040
	HM – Method	MMLE - I	0.5973	0.00401	0.02167	0.6013	0.0019
	AM – Method	MMLE - II	88.4966	0.10103	1.62597	88.5977	6.2966
	Median – Method	MMLE - II	0.6275	0.00352	0.02037	0.6310	0.0018*
	GM – Method	MMLE - II	0.7866	0.00723	0.04084	0.7938	0.0040
	HM – Method	MMLE - II	0.5973	0.00390	0.02053	0.6012*	0.0019

Table 2.6
Generalized Variances of Two-Step estimation at $\alpha = 4$

n	Step 1 of method of estimation α	Step 2 of method of estimation σ	Var($\boldsymbol{\alpha}$)	Var(σ)	Cov(α,σ)	Generalized Variances	
						I	II
5	AM – Method	MMLE - I	7178.78746	0.14240	7.49379	7178.9299	966.0871
	Median – Method	MMLE - I	16.23483	0.02833	0.20894	16.2632	0.4162
	GM – Method	MMLE - I	18.15355	0.03116	0.21304	18.1847	0.5204
	HM – Method	MMLE - I	8.67728	0.00773	-0.01627	8.6850	0.0668
	AM – Method	MMLE - II	7178.78746	0.14013	7.42319	7178.9276	950.8732
	Median – Method	MMLE - II	16.23483	0.02688	0.20041	16.2617	0.3963
	GM – Method	MMLE - II	18.15355	0.03063	0.20979	18.1842	0.5121
	HM – Method	MMLE - II	8.67728	0.00761	-0.01695	8.6849*	0.0658*
10	AM – Method	MMLE - I	1533.63812	0.10552	3.59242	1533.7436	148.9304
	Median – Method	MMLE - I	3.86319	0.01082	0.08090	3.8740	0.0352
	GM – Method	MMLE - I	4.78488	0.01608	0.12562	4.8010	0.0611
	HM – Method	MMLE - I	2.88123	0.00645	0.03553	2.8877	0.0173
	AM – Method	MMLE - II	1533.63812	0.10114	3.49243	1533.7393	142.9162
	Median – Method	MMLE - II	3.86319	0.01010	0.07551	3.8733	0.0333
	GM – Method	MMLE - II	4.78488	0.01540	0.12001	4.8003	0.0593
	HM – Method	MMLE - II	2.88123	0.00614	0.03248	2.8874*	0.0166*
15	AM – Method	MMLE - I	1023.19151	0.08761	3.80438	1023.2791	75.1667
	Median – Method	MMLE - I	2.02088	0.00691	0.05039	2.0278	0.0114
	GM – Method	MMLE - I	2.47425	0.01121	0.08256	2.4855	0.0209
	HM – Method	MMLE - I	1.69284	0.00539	0.03342	1.6982	0.0080
	AM – Method	MMLE - II	1023.19151	0.08372	3.68908	1023.2752	72.0557
	Median – Method	MMLE - II	2.02088	0.00654	0.04745	2.0274	0.0110
	GM – Method	MMLE - II	2.47425	0.01080	0.07924	2.4851	0.0204
	HM – Method	MMLE - II	1.69284	0.00518	0.03131	1.6980*	0.0078*
20	AM – Method	MMLE - I	575.7251	0.07170	2.41874	575.7968	35.4319
	Median – Method	MMLE - I	1.4229	0.00488	0.03774	1.4278	0.0055
	GM – Method	MMLE - I	1.7693	0.00852	0.06605	1.7778	0.0107
	HM – Method	MMLE - I	1.2843	0.00467	0.03273	1.2890	0.0049
	AM – Method	MMLE - II	575.7251	0.06865	2.34649	575.7938	34.0195
	Median – Method	MMLE - II	1.4229	0.00467	0.03571	1.4276	0.0054
	GM – Method	MMLE - II	1.7693	0.00825	0.06364	1.7776	0.0106
	HM – Method	MMLE - II	1.2843	0.00453	0.03106	1.2888*	0.0048*
25	AM – Method	MMLE - I	157.3273	0.06842	1.62624	157.3958	8.1200
	Median – Method	MMLE - I	1.1155	0.00371	0.03015	1.1192	0.0032*
	GM – Method	MMLE - I	1.3984	0.00720	0.05603	1.4056	0.0069
	HM – Method	MMLE - I	1.0618	0.00415	0.03083	1.0660	0.0035
	AM – Method	MMLE - II	157.3273	0.06565	1.58280	157.3930	7.8227
	Median – Method	MMLE - II	1.1155	0.00358	0.02864	1.1191	0.0032*
	GM – Method	MMLE - II	1.3984	0.00700	0.05417	1.4054	0.0069
	HM – Method	MMLE - II	1.0618	0.00404	0.02948	1.0659*	0.0034

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