Effect of Temperature on Vibrations of Elastic Circular Plate Under The Influence of Shear Effect

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Abstract

The computational study is presented here with object for design different structures by engineers used in many fields i.e. aircraft section, CD, missiles, mechanical and electrical device etc. Many structural plates have to operate under influence of temperature and have great importance in the engineering. This paper describes parabolic thermal vibrations of circular plates with foundation effect. PSS method has employed for determine the frequencies of vibrations for 4-modes using the functions based plates. The frequency parameter of the plate with elastically restrained edge conditions are presented for various values of foundation and temperature parameters. The obtain results have compare with available in the literature.

Key Words: - Young and shear modulus, thermal gradient, foundation parameter, frequency parameter.
1 Introduction

Tomar, Gupta and Jain have observed the axisymmetric vibrations of a non-homogeneous isotropic circular plate of linearly varying thickness taking an exponential variation of elastic modulus and density with radius vector [1]. Tomar et al are studied the vibrations of a circular plate of variable thickness on elastic foundation by various method with different boundary conditions [2]. For uniform circular plates, Mindlin and Deresiewicz has shown that for modirately thick plates the shearing effects cannot be neglected [3]. Gupta and sharma have analyzed the forced vibrations of a stepped rectangular beam with effect of shear and rotatory inertia [4]. Xiang and Reddy have studied the natural vibrations of rectangular plates with aninternal line hinge by leavy type solution and observed natural frequencies [5]. Lee has observed the free vibrations of plates by four node finite method under shear theory [6]. Lee and Cheng have observed the non linear free vibrations of orthotropic plates by differential quadrature method under shear effect [7]. In the recent years, thermal effect on solid bodies has highly increased because of rapid developments in space technology and high speed flights. In these problems, the thermal dependence of frequency of plates of different shapes is of great importance and designing many scientific devices. Tomar and Tiwari have studied the effect of linearly transient temperature field on frequencies of an isotropic circular plate of linearly varying thickness [8]. Recently, Tomar and Gupta considered the effect of harmonic and linearly temperature variations on axisymmetric vibrations of orthotropic circular plates of variable thickness respectively [9-10]. Thermal effect on axisymmetric vibrations of an orthotropic circular plate of uniform thickness has been considered by Tomer and Gupta [11]. Sarswat has analyzed the damped vibration of circular plate by Frobenius method with thermal effect [16]. This study observed the effect of the temperature distribution on the frequencies of an isotropic circular plate of non variable thickness with foundation effect. The equation of motion is analysis by PSS method and frequency parameters for first four vibration modes at clamped and simply-supported edges have been computed. The numerical results are shown in graphical form. The present investigations are helpful in designing many scientific devices where uniform structure are use under high intensity
heat fluxes and properties are significant change in vibrations.

2 Analysis

Temperature Variation
The plate material is considered to be subjected to a parabolic temperature distribution (in R-direction) is given by

\[ T = T_0 \left( 1 - R^2 \right) \]

Where \( T \) and \( T_0 \) denotes the temperature excess above the reference temperature at any point \( R \) and at the end \( R = 0 \) respectively. The temperature dependence of the modulus of elasticity for most of the engineering materials is given by [12-14, 17], therefore, one can have

\[ \bar{E}(T) = \bar{E}_0 \left( 1 - \xi T \right) \text{ and } \bar{G}(T) = \bar{G}_0 \left( 1 - \xi T \right) \]

Where \( \bar{E}_0, \bar{G}_0 \) are values of Young’s and Shear, modulus respectively, at \( T = 0 \) and \( \xi \) is a constant.

\[ \bar{E}(R) = \bar{E}_0 \{ 1 - \eta \left( 1 - R^2 \right) \} \text{ and } \bar{G}(R) = \bar{G}_0 \{ 1 - \eta \left( 1 - R^2 \right) \} \]

Where, \( = \xi T_0, \left( 0 \leq \eta \leq 1 \right) \) is called the thermal gradient and is a arbitrary constant.

Equation of Motion
The governing equation of motion for non-dimensional variables is given by

\[ \left( H^2 \bar{E}' \right) \left[ \Psi' + \left( \frac{\nu}{R} \right) \Psi \right] + \left( H^2 \bar{E} \right) \left[ \Psi'' + \left( \frac{1}{R} \right) \Psi' - \left( \frac{1}{R} \right) \Psi \right] - 12 \left( 1 - \nu^2 \right) R^2 \bar{G} \left[ \Psi + \Psi' \right] + \left[ A_0 a^2 \rho H^2 \right] \Psi = 0 \]

And

\[ W = - \left[ \frac{1}{12 (1 - \nu^2) \left( p^2 a \rho - \left( k_0 \right) \right)} \right] \left[ \left\{ H^2 \bar{E} \right\} \Psi'' \right. + \left\{ \left( 2H^2 \bar{E}' \right) + \left( \frac{2}{R} \right) \left( H^2 \bar{E} \right) \right\} \Psi' \]
\[
+ \left\{ \left( H^2E^{II} \right) + \left( \frac{2 + \nu}{R} \right) \left( H^2E^{I} \right) - \left( \frac{1}{R^2} \right) \left( H^2E \right) + \left( A_0 \bar{\rho} H^2a^2 \right) \right\} \Psi^{I} + \\
\left\{ \left( \frac{\nu}{R} \right) \left( H^2E^{II} \right) - \left( \frac{1}{R^2} \right) \left( H^2E^{I} \right) + \left( \frac{1}{R^3} \right) \left( H^2E \right) + \left( A_0 \bar{\rho} H^2a^2 \right) \left( \frac{1}{R} \right) \right\} \Psi 
\]

Where,

\[
R = \frac{r}{a}, \ \overline{W} = \frac{w}{a}, \ E = \frac{E}{a}, \ \overline{G} = \frac{G}{a}, \ \overline{\rho} = \frac{\rho}{a}, \ H = \frac{h}{a},
\]

\(a\) is the radius of the plate and prime denotes the derivative with respect to \(R\). Let

\[
\overline{W} = \frac{w}{a}
\]

with considering the mass during of the plate materials are constants.

Imposing relation (3) on equation (4) and (5), one obtained the equation of motion as

\[
\overline{W} = \left[ \frac{A_1}{A_2} \right] \left[ B_1^{(1)} \right] \Psi^{III} + \left\{ B_2^{(1)} + \left( \frac{B_2^{(2)}}{R} \right) \right\} \Psi^{I} + \\
\left\{ B_3^{(1)} + \left( \frac{B_3^{(2)}}{R} \right) + \left( \frac{B_3^{(3)}}{R^2} \right) \right\} \Psi + \left\{ \left( \frac{B_4^{(1)}}{R} \right) + \left( \frac{B_4^{(2)}}{R^2} \right) \right\} \Psi 
\]

and

\[
\left[ T_1^{(1)} + R^2T_1^{(2)} + R^4T_1^{(3)} \right] \Psi^{IV} + \left[ \frac{T_2^{(1)}}{R} + RT_2^{(2)} + R^3T_2^{(3)} \right] \Psi^{III} + \\
+ \left[ \frac{T_3^{(1)}}{R^2} + T_3^{(2)} + R^2T_3^{(3)} \right] \Psi^{II} + \\
+ \left[ \frac{T_4^{(1)}}{R^3} + \frac{T_4^{(2)}}{R} + RT_4^{(3)} \right] \Psi 
\]

\[
+ \left[ \frac{T_5^{(1)}}{R^4} + \frac{T_5^{(2)}}{R^2} + T_5^{(3)} + R^2T_5^{(4)} \right] \Psi = 0
\]

Where,

\[
A_1 = \left( -\frac{H^2}{12} \right), \ A_2 = (\Omega^2 - D_K/C_0H),
\]
\[ A_3 = (\Omega^2 (1 + C_0) - D_K/H), \quad A_4 = (\Omega^2 C_0 - D_K/H) \]

\[ B_2^{(1)} = (1 - \eta + \eta R), \quad B_3^{(1)} = (4 \eta), \quad B_2^{(2)} = 2 (1 - \eta), \quad B_3^{(1)} = \Omega^2, \]

\[ B_3^{(2)} = \eta (1 + \nu), \quad B_3^{(3)} = - (1 - \eta), \quad B_3^{(4)} = \Omega^2, \quad B_2^{(2)} = (1 - \eta), \]

\[ C_0 = \left( \frac{E_0}{(1 - \eta^2) G_0 K^2} \right), \quad D_K = \left( \frac{K F}{G_0 K^2} \right), \quad \Omega^2 = \left\{ \frac{(1 - \nu^2) \rho a^2 p^2}{E_0} \right\} \]

\[ T_1^{(1)} = (1 - \eta^2), \quad T_1^{(2)} = (2 \eta (1 - \eta)), \quad T_1^{(3)} = \eta^2, \quad T_2^{(1)} = 2 (1 - \eta^2), \]

\[ T_2^{(2)} = 8 \eta^2, \quad T_2^{(3)} = 10 \eta (1 - \eta), \quad T_3^{(1)} = -3 (1 - \eta^2), \]

\[ T_3^{(2)} = [\eta (1 - \eta) (11 + 2 \nu) + (A_3 - 3 \eta) (1 - \eta)], \quad T_3^{(3)} = (\eta^2 (11 + 2 \nu) + A_3 \eta), \]

\[ T_4^{(1)} = 3 (1 - \eta^2), \quad T_4^{(2)} = (1 - \eta) [2 \eta (\nu + 1) + A_3], \quad T_4^{(3)} = [\eta^2 (2 \nu - 1) + A_3 \eta], \]

\[ T_5^{(1)} = 3 (1 - \eta^2), \quad T_5^{(2)} = (1 - \eta) [3 \eta - (\eta (2 \nu - 1) + A_3)], \quad T_5^{(3)} = 2 \eta \nu + A_3 \]

\[ + A_5 (1 - \eta) (A_3 - \Omega^2) - [\eta^2 (2 \nu - 1) + A_3 \eta], \quad T_5^{(4)} = [\eta A_5 A_4] \]

\[ A_5 = -\frac{12 C_0 H^2}{E_0 \rho a^2 p^2} \]

Hence, \( D_K \) is foundation modulus parameter, \( p \) is circular frequency and is frequency parameter.

### 3 Solution and its Convergence

Let the solution for \( \bar{W} \) has been assumed in the series from as:

\[ \bar{W} = \sum_{\lambda=0}^{\infty} a_\lambda R^{C+\lambda} \quad with \quad a_0 \neq 0 \]

Where \( C \) is the exponent of singularity, Using equation (8) (7) one obtains

\[ \sum_{\lambda=0}^{\infty} a_\lambda \left[ \begin{array}{c} b_\lambda^{(3)} L_1^{(1)} + b_\lambda^{(2)} L_2^{(1)} + b_\lambda^{(1)} L_3^{(1)} + b_\lambda L_4^{(1)} + L_5^{(1)} \\ b_\lambda^{(2)} L_1^{(2)} + b_\lambda^{(1)} L_2^{(2)} + b_\lambda L_3^{(2)} + b_\lambda L_4^{(2)} + L_5^{(2)} \end{array} \right] R^{C+\lambda-4} \]

\[ + \sum_{\lambda=0}^{\infty} a_\lambda \left[ \begin{array}{c} b_\lambda^{(3)} L_1^{(2)} + b_\lambda^{(2)} L_2^{(2)} + b_\lambda^{(1)} L_3^{(2)} + b_\lambda L_4^{(2)} + L_5^{(2)} \end{array} \right] R^{C+\lambda-3} \]
\begin{align*}
+ \sum_{\lambda=0}^{\infty} a_{\lambda} \left[ b_{\lambda}^{(3)} L_{1}^{(3)} + b_{\lambda}^{(2)} L_{2}^{(3)} + b_{\lambda}^{(1)} L_{3}^{(3)} + b_{\lambda} L_{4}^{(3)} + L_{5}^{(3)} \right] R^{C+\lambda-2} \\
+ \sum_{\lambda=0}^{\infty} a_{\lambda} \left[ b_{\lambda}^{(1)} L_{3}^{(4)} + b_{\lambda} L_{4}^{(4)} + L_{5}^{(4)} \right] R^{C+\lambda-1} + \sum_{\lambda=0}^{\infty} a_{\lambda} \left[ L_{5}^{(5)} \right] R^{C+\lambda} + \\
\sum_{\lambda=0}^{\infty} a_{\lambda} \left[ L_{5}^{(6)} \right] R^{C+\lambda+1} = 0
\end{align*}

Respectively, where

\begin{align*}
b_{\lambda}^{(3)} &= (C + \lambda) (C + \lambda - 1) (C + \lambda - 2) (C + \lambda - 3), \quad b_{\lambda}^{(2)} = (C + \lambda) (C + \lambda - 1) (C + \lambda - 2), \\
b_{\lambda}^{(1)} &= (C + \lambda) (C + \lambda - 1), \quad b_{\lambda} = (C + \lambda), \\
L_{1}^{(1)} &= (1-\eta^{2}), \quad L_{2}^{(1)} = 2(1-\eta^{2}), \quad L_{3}^{(1)} = -3(1-\eta^{2}), \quad L_{4}^{(1)} = 3(1-\eta^{2}), \quad L_{5}^{(1)} = -3(1-\eta^{2}) \\
L_{1}^{(2)} &= 2\eta(1-\eta), \quad L_{2}^{(2)} = 10\eta(1-\eta), \quad L_{3}^{(2)} = (8\eta+2\eta\nu+A_{3}), \quad L_{4}^{(2)} = \eta^{2}, \\
L_{2}^{(3)} &= 2\eta\nu, \quad L_{3}^{(3)} = (2\eta(1+\nu)+A_{3}) = -L_{5}^{(2)}, \quad L_{4}^{(3)} = (\eta^{2}(11+2\nu)+A_{3}\eta), \\
L_{4}^{(3)} &= (\eta^{2}(2\nu-1)+A_{3}\eta+2\eta A_{4}), \quad L_{5}^{(3)} = -\eta^{2}(2\nu-1)+A_{3}\eta)+A_{4}(2\eta\nu+\Omega^{2}) \\
-A_{5}A_{4}(1-\eta), \quad L_{1}^{(4)} &= (A_{5}A_{4}\eta)
\end{align*}

The Indicial roots are obtained for equating to zero the coefficient of the lowest power of \( R \) is \( C = 3,1,1,-1 \). For next higher power of \( R \), for \( C = 1 \) this gives \( a_{1} = 0 \). For RC-2, \( a_{2} (C =1) \) is indeterminate and taken as constant with \( a_{0} \). For other higher powers of \( R \), \( a_{3} \) to \( a_{5} \) is obtained in terms of \( a_{0} \) and \( a_{2} \). The remaining unknown constants \( a_{\lambda} (\lambda = 6,7,...) \) are determined from the recurrence relation

\begin{align*}
a_{\lambda+6} \left[ B_{1} L_{1}^{(1)} \right] + a_{\lambda+4} \left[ B_{2} L_{1}^{(2)} + B_{3} L_{3}^{(2)} + B_{4} L_{4}^{(2)} \right] + \\
 a_{\lambda+2} \left[ B_{5} L_{1}^{(3)} + B_{6} L_{3}^{(3)} + B_{7} L_{4}^{(3)} + L_{5}^{(3)} \right] + a_{\lambda} \left[ L_{1}^{(4)} \right] = 0
\end{align*}

where

\begin{align*}
B_{1} &= (C + \lambda + 6)^{2}(C + \lambda + 8)(C + \lambda + 4), \quad B_{2} = (C + \lambda + 7)(C + \lambda + 4) \\
(C + \lambda + 5)(C + \lambda + 3), \quad B_{3} = (C + \lambda + 4)(C + \lambda + 5), \quad B_{4} = (C + \lambda + 4) \\
B_{5} &= (C + \lambda + 8)(C + \lambda + 3)(C + \lambda + 2)(C + \lambda + 1)
\end{align*}
If the notations:
\[ a_\lambda = F_\lambda a_0 + G_\lambda a_2 \text{ with } \lambda = 0, 1, 2, 3, 4 \ldots \]
are introduced, the solution \( \Psi \) at \( C=1 \) can be written as
\[ \Psi = a_0 X(R, \Omega) + a_2 Y(R, \Omega) \]

Where,
\[
\begin{align*}
X(R, \Omega) &= R + \sum_{\lambda=1}^{\infty} F_\lambda R^{\lambda+1} \\
Y(R, \Omega) &= \sum_{\lambda=1}^{\infty} G_\lambda R^{\lambda+1}
\end{align*}
\]

where \( F_\lambda \) and \( G_\lambda \) are function of \( \eta, D_k, \Omega \), and \( C_0 \). It is observed that no new solution will arise if \( C = 3 \) and already contained in (11) with \( a_0 \) and \( a_2 \). The solution corresponding to \( C = 1 \) and -1 of the indicial equation being singular at \( R = 0 \) are omitted, because they are of no interest to anyone. The solution (11) and (13) is uniformly convergent in the interval \( 0 \leq R \leq 1 \) for all \( | \mu < 1 | \) according to Lamb technique [15]. Hence the solution is convergent for all \( \pm \left( \frac{\eta}{1-\eta} \right)^{1/2} < 1 \) since \( \overline{\Psi} \) is expressed in terms of \( \overline{W} \) and its derivatives, the solution for \( \overline{\Psi} \) is also convergent.

\[ \overline{W} = a_0 U_1(R, \Omega) + a_2 U_2(R, \Omega) \]

where,
\[
\begin{align*}
U_1(R, \Omega) &= -\left( \frac{A_1}{A_2} \right) \left[ B_1^{(1)} X^{III} + \left( B_2^{(1)} + \frac{B_2^{(2)}}{R} \right) X^{II} \right] + \left( B_3^{(1)} + \frac{B_3^{(2)}}{R} + \frac{B_3^{(3)}}{R^2} \right) X + \left( B_4^{(1)} + \frac{B_4^{(2)}}{R} \right) X \\
U_2(R, \Omega) &= -\left( \frac{A_1}{A_2} \right) \left[ B_1^{(1)} Y^{III} + \left( B_2^{(1)} + \frac{B_2^{(2)}}{R} \right) Y^{II} \right] + \left( B_3^{(1)} + \frac{B_3^{(2)}}{R} + \frac{B_3^{(3)}}{R^2} \right) Y + \left( B_4^{(1)} + \frac{B_4^{(2)}}{R} \right) Y
\end{align*}
\]
Where a prime denotes the derivative with respect to R. This solution for is non-singular at all points of the plate because the substitution for X and Y cancels R from the denominator in the terms involving 1/R, 1/R2 and 1/R3

4 Boundary Conditions and Frequency Equations

Now employing the following boundary conditions and frequency equations of circular plates have been obtained as:

**Clamped Plate:**

Now plate clamped at \( r = a \), the \( \bar{W} \) and rotation of the plate element at the edge should be zero.

\[
\bar{W} \bigg|_{R=1} = 0 = \psi \bigg|_{R=1}
\]

Imposing (15) to (11) and (13), one gets after eliminating \( a_{0} \) and \( a_{2} \), the frequency equation for clamped-plate as :

\[
\begin{vmatrix}
X(1, \Omega) & Y(1, \Omega) \\
U_{1}(1, \Omega) & U_{2}(1, \Omega)
\end{vmatrix}
= 0
\]

Where,

\[
X(1, \Omega)Y(1, \Omega)U_{1}(1, \Omega)
\]

and

\[
U_{2}(1, \Omega)
\]

are given by equations (12) and (14) respectively:

**Simply Supported Plates:**

Now plate at the edge \( r = a \), then \( \bar{W} \) and Mr at this edge, should be zero

\[
\bar{W} \bigg|_{R=1} = 0 = \left[ \psi^{1} + \frac{\nu}{R} \psi \right] \bigg|_{R=1}
\]

Again using (11) and (13) one obtains the frequency equation for this boundary condition, the plate as : i.e.

\[
\begin{vmatrix}
U_{1}(1, \Omega) & U_{2}(1, \Omega) \\
U_{3}(1, \Omega) & U_{4}(1, \Omega)
\end{vmatrix}
= 0
\]
where,

\[
U_3 (1, \Omega) = \left[ \frac{\partial X (R, \Omega)}{\partial R} + \frac{\nu}{R} X (R, \Omega) \right] \bigg|_{R=1}
\]

and

\[
U_4 (1, \Omega) = \left[ \frac{\partial Y (R, \Omega)}{\partial R} + \frac{\nu}{R} Y (R, \Omega) \right] \bigg|_{R=1}
\]

5 Results and Discussion

The expressions (16) and (18) for frequency parameters are transcendental equations in from which infinitely many roots can be observed. Here the frequencies of first 4 modes of vibrations of a clamped and simply - supported circular plates have been computed for different values of \( \eta \) and \( D_k \) using computer programming but first 2 modes of vibrations presented here in graphically form . In all the cases the values of \( \eta = 0.4 \) and \( k^2 = 0.85 \) have been considered here. While calculating the frequencies terms with absolute value greater than \( 10^{-8} \) in the series of and \( \psi \) retained. The results according to figure 1 shows the thermally effected variation of \( \omega \) parameters with \( D_k \) when the shear theory can be considered. Observed from figure the values of \( \omega \) increases with the increase of \( D_k \) for all presented modes of vibration , here for both conditions of the heated and unheated \( (\eta=0) \) circular plates. It is also noted that, the \( \omega \) for clamped plate are greater than the corresponding \( \omega \) for simply-supported plate and the frequencies of heated plated are lower than that of unheated plated \( (\eta=0) \) in both the edges condition. The figure 2 represent the thermal effect on the vibrations of a circular plate of uniform thickness with modulus \( D_k \). It is observed that in both boundary conditions, the \( \omega \) decreases with the increase of \( \eta \) for non-homogeneous materials of the plate. One can see that the frequencies of heated plate are greater than that of unheated \( (\eta =0) \). Also, the value of \( \omega \) in shear theory is less than the corresponding \( \omega \) in classical plate theory. Thus, the effects of shear and rotatory inertia cannot be neglected for thin plates, when vibrations corresponding to the higher modes are considered.
References


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