

Similarity Measures of Pentagonal Fuzzy Numbers

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Abstract

Fuzzy number is a tool to characterize the impreciseness captured due to the presence of ill-defined terms in the natural language statements. In this paper, we made an extensive analysis of the similarity relation between two pentagonal fuzzy numbers. Also, we introduce various similarity measures between two pentagonal fuzzy numbers along with its geometrical illustration.

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1 Introduction

In the year 1965, Lotfi A. Zadeh [13] an electrical engineer, made an extensive empirical investigation to study the intangible real life phenomenon. This analysis made him to extend the classical, categorical set theoretical concepts into fuzzy, graded set theoretical concepts. Zadeh initiated this study to quantify the human subjective information which modern day computer fails to process. Fuzzy sets provides a

fundamental and concise theoretical framework for processing and reasoning the humans rational subjectivities. Human perceptions do get computed with the help of fuzzy sets more specifically with the support obtained from fuzzy numbers. In addition to that, concepts such as type-2 fuzzy sets, intuitionistic fuzzy sets, hesitant fuzzy sets, fuzzy multisets and fuzzy numbers [6] are also employed to represent the imprecision. Richness of complexity in the natural language statements and incomplete information due to ill defined linguistic terms are accommodated through fuzzy numbers. Fuzzy number gets its extension from the real number. Representing linguistic variables in terms of fuzzy numbers helps to quantify the subjective information. For instance, the term "exactly 5" can easily be represented by the classical set which denotes the real number 5. Whereas linguistic terms like "around 5", "nearly 5", "approximately 5", "roughly 5", "more or less 5", "about 5" are difficult to characterize through classical sets. Circumstances such as the one explained above made the need of fuzzy number very essential. Triangular, trapezoidal and bell shaped fuzzy numbers [6] are the commonly used fuzzy number to represent the incomplete vaguely defined linguistic terms.

Pattern recognition is one such field where fuzzy sets are widely used to resolve the imprecise nature of vague boundaries among the patterns [2]. During the classification process, it is very common that many of the patterns that we tried to categorize may have some similar characteristics. Fuzzy membership functions such as triangular, trapezoidal and bell shaped functions are elaborately used to quantify and classify the noisy data in pattern recognition and clustering techniques. On classifying the massive information and grouping them together involves huge computational work. Finding the degree of similarity between the patterns is very essential in order to reduce the computation. Fuzzy similarity measures are predominantly employed to classify the resemblance between patterns.

Various similarity measures [5] have been introduced based on the concepts such as geometric distance [4], l_p -metric [3], graded mean integration [1], center of gravity [8], perimeter [10] and cosine angular distance. Through this paper, we made an extensive study on similarity relation between two pentagonal fuzzy numbers. T. Pathinathan and K. Ponnivalavan introduced pentagonal fuzzy number [11] in the year 2014. Also they developed the generalized notion of pentagonal fuzzy number [12] in the year 2015 along with the set theoretic operations. In addition to that they established the concept of area, centroid and median [12] of the pentagonal fuzzy number with geometrical illustration. Rajkumar

and T. Pathinathan [7] used pentagonal fuzzy number to sieve out the poor in the Nalanda District, Bihar. The main objective of this paper is to brief out the conceptual theory behind the pentagonal fuzzy number with the help of real life phenomenon. In this paper, we propose a various kinds of similarity measures between two pentagonal fuzzy numbers. Also we have calculated the graded mean integration, perimeter for pentagonal fuzzy number. Throughout this paper, we adopt $\tilde{A}$ notation for representing the fuzzy set, whereas in our previous papers [11] [12] [7] we have used $\sim$ below the letter A .

The paper is organized as follows. Section two presents the basic preliminaries and definitions on fuzzy sets and fuzzy numbers. Section three gives a brief note on conceptual background of pentagonal fuzzy number with illustration. Section four introduces the various concepts of similarity measures between two pentagonal fuzzy numbers followed by conclusion in the section five.

2 Basic Preliminaries

In this section, we present some basic definitions and concepts of fuzzy sets and fuzzy numbers.

2.1 Fuzzy Set

A *fuzzy set* is characterized by a membership function mapping the elements of a domain to the unit interval $[0, 1]$. A fuzzy set $\tilde{A}$ of X is defined by the following pair, such as

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) / x \in X\} \quad (1)$$

2.2 Fuzzy Number (FN)

A *fuzzy number* $\tilde{A}$ is defined as $\tilde{A} = (a_1, a_2, a_3)$ where a_1, a_2, a_3 are real numbers with $a_1 \leq a_2 \leq a_3$ and the membership function is expressed by,

$$F_{\tilde{A}}(x) = \begin{cases} F_{\tilde{A}}^l(x) & a_1 \leq x \leq a_2 \\ F_{\tilde{A}}^r(x) & a_2 \leq x \leq a_3 \end{cases}$$

Also, $F_{\tilde{A}}^l(x)$ and $F_{\tilde{A}}^r(x)$ are the two piecewise continuous function connected at the maximum (core).

Triangular, trapezoidal and bell shaped fuzzy numbers are the most commonly used fuzzy number to represent the ill-defined linguistic terms [6].

3 Pentagonal Fuzzy Number (PFN)

Any real phenomenon involves rational elements are of qualitative (subjective) in nature. The existing fuzzy numbers such as triangular and trapezoidal fuzzy number [6] has a kind of strictly increasing and decreasing curve (usually a linear type of representation) in both the left and right side of the membership function. And in many cases, the triangular and trapezoidal fuzzy number representations are found to be insufficient in capturing the preciseness of the uncertainty. In most of the cases, it is impossible to represent the ill-defined terms by the either an increasing or by decreasing function. Sometimes the curve which we have seen in the traditional fuzzy number does not capture the complete reality into an account. So this kind of situation need some replacement and alternative. T. Pathinathan and K. Ponnivalavan introduced a new type of fuzzy number called pentagonal fuzzy number [11] [12] as an alternative to the situation where the curve has variations in the α level.

A *pentagonal fuzzy number* $\tilde{A}_P$ is defined as $\tilde{A}_P = (a_1, a_2, a_3, a_4, a_5)$ where a_1, a_2, a_3, a_4, a_5 are the real numbers and its membership function is given by,

$$F_{\tilde{A}}(x) = \begin{cases} 0 & x < a_1 \\ \frac{(x-a_1)}{(a_2-a_1)} & a_1 \leq x \leq a_2 \\ \frac{1}{2} \frac{(x-a_2)}{(a_3-a_2)} & a_2 \leq x \leq a_3 \\ 1 & x = a_3 \\ \frac{1}{2} \frac{(a_4-x)}{(a_4-a_3)} & a_3 \leq x \leq a_4 \\ \frac{(a_5-x)}{(a_5-a_4)} & a_4 \leq x \leq a_5 \\ 0 & x > a_5 \end{cases}$$

with $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5$.

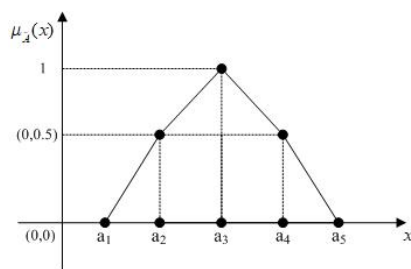


Figure 1: Pentagonal Fuzzy Number

Through this work, we tried to introduce the concepts related with measuring similarity between two pentagonal fuzzy numbers.

4 Similarity measures for PFNs

In many pattern recognition and image processing problems, it is necessary to find the similarity between the imprecise objects in order to classify them into various groups. Fuzzy number is a tool to define the subjective impreciseness numerically. So, finding the similarity measure between fuzzy numbers is very important to reduce the complexity in any pattern recognition problems [8] [9]. This section discusses the similarity measure between two pentagonal fuzzy numbers and we introduce four types of similarity measures for pentagonal fuzzy number.

4.1 Similarity measure between two pentagonal fuzzy numbers based on geometric distance

Let $\tilde{A}_P = (a_1, a_2, a_3, a_4, a_5)$ and $\tilde{B}_P = (b_1, b_2, b_3, b_4, b_5)$ be the two pentagonal fuzzy numbers where $a_1, a_2, a_3, a_4, a_5, b_1, b_2, b_3, b_4, b_5$ are the real numbers with $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5$ and $b_1 \leq b_2 \leq b_3 \leq b_4 \leq b_5$. The similarity measure $S_1(\tilde{A}, \tilde{B})$ between two pentagonal fuzzy number based on the geometric distance is given by the formula:

$$S_1(\tilde{A}_P, \tilde{B}_P) = 1 - \frac{\sum_{i=1}^5 |a_i - b_i|}{5} \quad (2)$$

4.2 Similarity measure based on Lee's Optimal Aggregation Method [3]

The similarity measure $S_2(\tilde{A}_P, \tilde{B}_P)$ between two pentagonal fuzzy number $\tilde{A}_P = (a_1, a_2, a_3, a_4, a_5)$ and $\tilde{B}_P = (b_1, b_2, b_3, b_4, b_5)$ based on Lee's optimal aggregation method is defined by the formula:

$$S_2(\tilde{A}_P, \tilde{B}_P) = 1 - \frac{\|\tilde{A}_P - \tilde{B}_P\|_{l_p}}{\|U\|} \times 5^{-\frac{1}{p}} \quad (3)$$

where $\|\tilde{A}_P - \tilde{B}_P\|_{l_p} = (\sum_{i=1}^5 |a_i - b_i|^p)^{\frac{1}{p}}$ and $\|U\| = \max U - \min U$ in which U is the universe of discourse and p is a positive integer, Here $p=1$, since we have the pentagonal fuzzy number in l_1 metric space. So the above equation 3 is reduced into the following form such as:

$$S_2(\tilde{A}_P, \tilde{B}_P) = 1 - \frac{|\tilde{A}_P - \tilde{B}_P|_{l_1}}{|U|} \times 5^{-1} \quad (4)$$

4.3 Similarity measure using graded mean integration representation

The graded mean integration representation for pentagonal fuzzy number $\tilde{A}_P = (a_1, a_2, a_3, a_4, a_5)$ is defined as follows:

$$P(\tilde{A}_P) = \frac{a_1 + 4a_2 + 2a_3 + 4a_4 + a_5}{6} \quad (5)$$

The graded mean integration representation for generalized pentagonal fuzzy number is calculated as follows:

$$P(\tilde{A}) = \frac{a_1(5 - 4\sqrt{w}) + a_2(1 + 4\sqrt{w}) + 8a_3 + a_4(9 + 4\sqrt{w}) + a_5(5 - 4\sqrt{w})}{10} \quad (6)$$

The similarity measure $S_3(\tilde{A}_P, \tilde{B}_P)$ between two pentagonal fuzzy number $\tilde{A}_P = (a_1, a_2, a_3, a_4, a_5)$ and $\tilde{B}_P = (b_1, b_2, b_3, b_4, b_5)$ based on graded mean integration representation is given by the formula:

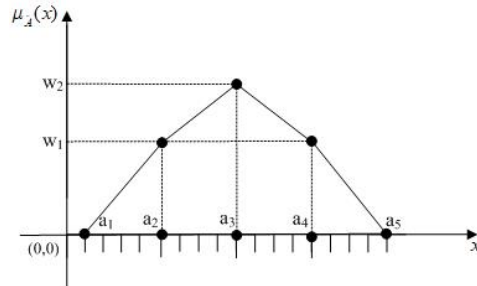
$$S_3(\tilde{A}_P, \tilde{B}_P) = \frac{1}{1 + d(\tilde{A}, \tilde{B})} \quad (7)$$

where $d(\tilde{A}, \tilde{B}) = |P(\tilde{A}_P) - P(\tilde{B}_P)|$, with $P(\tilde{A}_P) = \frac{a_1 + 4a_2 + 2a_3 + 4a_4 + a_5}{6}$ and $P(\tilde{B}_P) = \frac{b_1 + 4b_2 + 2b_3 + 4b_4 + b_5}{6}$.

4.4 Similarity measure using geometric distance, perimeter and height of the two pentagonal fuzzy numbers

4.4.1 Perimeter of a pentagonal fuzzy number

Let $\tilde{A}_P = (a_1, a_2, a_3, a_4, a_5; w_1, w_2)$ be the generalized pentagonal fuzzy number where a_1, a_2, a_3, a_4, a_5 are the real numbers with $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5$. w_1 and w_2 be the two different α -level in which w_1 be the level where the variation takes place between two piecewise continuous curves and w_2 be the maximum core value of the generalized pentagonal fuzzy number.

Figure 2: Pentagonal Fuzzy Number $\tilde{A}_P$

In the above figure 2, (a_2, w_1) be the point where the bounded function have small deviation from the increasing monotonic condition and reaches the maximum core at (a_3, w_2) . Whereas in triangular fuzzy number, the function has strictly monotonic increasing curve at the left side of the function which reaches the maximum core at a_2 and takes strictly decreasing phase at the right side of the function. Then the perimeter of the generalized pentagonal fuzzy number shown in the figure 2 is calculated as follows:

$$P(\tilde{A}_P) = \sqrt{(a_2 - a_1)^2 + w_1^2} + \sqrt{(a_3 - a_2)^2 + (w_2 - w_1)^2} + \sqrt{(a_4 - a_3)^2 + (w_1 - w_2)^2} + \sqrt{(a_5 - a_4)^2 + w_1^2} + (a_5 - a_1) \quad (8)$$

Based on the approach introduced by Wei and Chen [10], the similarity measure combines the concepts of geometric distance defined in the equation 2 and perimeter calculated in the equation 8 with the height of the generalized pentagonal fuzzy number. Then the similarity measure between $\tilde{A}_P = (a_1, a_2, a_3, a_4, a_5; w_1, w_2)$ and $\tilde{B}_P = (b_1, b_2, b_3, b_4, b_5; w'_1, w'_2)$ is defined by the formula:

$$S_4(\tilde{A}_P, \tilde{B}_P) = \left(1 - \frac{\sum_{i=1}^5 |a_i - b_i|}{5}\right) \times \frac{\min(P(\tilde{A}_P), P(\tilde{B}_P)) + \min(w_2, w'_2)}{\max(P(\tilde{A}_P), P(\tilde{B}_P)) + \max(w_2, w'_2)} \quad (9)$$

$$\begin{aligned} \text{where } P(\tilde{A}_P) &= \sqrt{(a_2 - a_1)^2 + w_1^2} + \sqrt{(a_3 - a_2)^2 + (w_2 - w_1)^2} \\ &+ \sqrt{(a_4 - a_3)^2 + (w_1 - w_2)^2} + \sqrt{(a_5 - a_4)^2 + w_1^2} + (a_5 - a_1) \text{ and} \\ P(\tilde{B}_P) &= \sqrt{(b_2 - b_1)^2 + w_1'^2} + \sqrt{(b_3 - b_2)^2 + (w_2' - w_1')^2} \\ &+ \sqrt{(b_4 - b_3)^2 + (w_1' - w_2')^2} + \sqrt{(b_5 - b_4)^2 + w_1'^2} + (b_5 - b_1). \end{aligned}$$

4.5 An illustrative example 1

Let $\tilde{A}_P = (0.1, 0.2, 0.3, 0.4, 0.5)$ and $\tilde{B}_P = (0.2, 0.3, 0.4, 0.5, 0.6)$ be the two pentagonal fuzzy number with an increasing order. The figure 3 shows the geometrical representation of pentagonal fuzzy number $\tilde{A}_P$ and $\tilde{B}_P$.

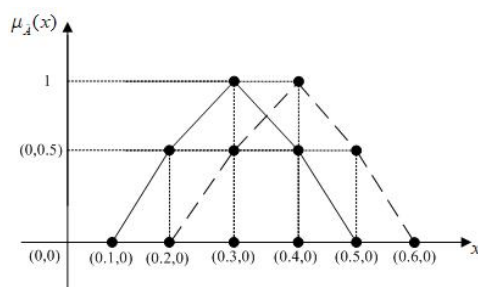


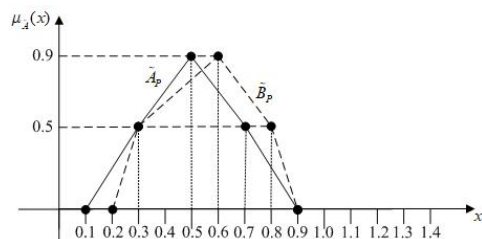
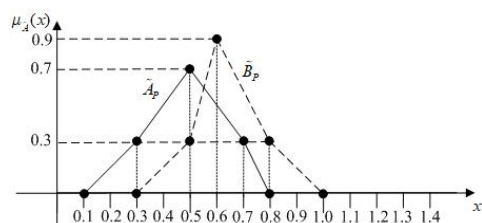
Figure 3: Pentagonal Fuzzy Number $\tilde{A}_P$ and $\tilde{B}_P$

Then the similarity measure defined in the equations 2, 4 and 7 is calculated as follows: $S_1(\tilde{A}_P, \tilde{B}_P) = 0.9$, $S_2(\tilde{A}_P, \tilde{B}_P) = 0.8$ and $S_3(\tilde{A}_P, \tilde{B}_P) = 0.8333$. The above values show the three different similarity measure values between the two pentagonal fuzzy numbers. In this example, we have considered the maximum core value of the two pentagonal fuzzy number to be 1. And we have calculated the three different similarity measures, which shows a minor variations in the degree of similarity between the normalized pentagonal fuzzy numbers.

4.6 An illustrative example 2

In this illustration, we have considered two sets of pentagonal fuzzy numbers with variation in both w_1 and w_2 level (Section 4.4.1).

SET A Let $\tilde{A}_P = (0.1, 0.3, 0.5, 0.7, 0.9; 0.5, 0.9)$ and $\tilde{B}_P = (0.2, 0.3, 0.6, 0.8, 0.9; 0.5, 0.9)$ be the two pentagonal fuzzy number with an increasing order and has α -level at 0.5 and core at 0.9. Then the similarity measure between the pentagonal fuzzy numbers is calculated by using equation 9 as follows: $S_4(\tilde{A}_P, \tilde{B}_P) = 0.6815$.

Figure 4: Pentagonal Fuzzy Number $\tilde{A}_P$ and $\tilde{B}_P$ (SET A)Figure 5: Pentagonal Fuzzy Number $\tilde{A}_P$ and $\tilde{B}_P$ (SET B)

SET B Let $\tilde{A}_P = (0.1, 0.3, 0.5, 0.7, 0.8; 0.3, 0.7)$ and $\tilde{B}_P = (0.3, 0.5, 0.6, 0.8, 1; 0.3, 0.9)$ be the two pentagonal fuzzy number with an increasing order and has α -level at 0.3 and core at 0.7 and 0.9 respectively. Then the similarity measure between the pentagonal fuzzy numbers is calculated by using equation 9 as follows: $S_4(\tilde{A}_P, \tilde{B}_P) = 0.7007$.

5 Conclusion

In this paper, we have introduced similarity measures between two pentagonal fuzzy numbers based on geometric distance, l_p -metric distance, graded mean integration representation and perimeter of a pentagonal fuzzy number. Also we provided the numerical illustration for the similarity measure between two pentagonal fuzzy numbers.

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