POWER SPECTRUM ESTIMATION OF SEISMIC WAVE USING PERIODOGRAM METHOD

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Abstract

Volcanoes, Tsunamis and many other seismic sources produce sudden elastic waves that propagate through the Earth. The time of occurrence and the frequencies of these earthquake waves must be predicted to mitigate the damage. These parameters are attained from the power spectrum. In this article, Periodogram technique is used for the calculation of spectral density. Simulation is carried out and the results are presented.

Keywords: Adaptive signal processing, Stochastic signal processing, Applied statistics, Precise estimation technique, Seismology.

1. Introduction
1.1. Seismology

Rocks beneath the earth’s surface do not reside only in one particular direction. When rocks underneath the faults slide against each other and due to the tension among the rocks, energy is released along the earth’s surface in form of seismic waves [1]. Landslides, volcanoes, earthquakes discharge seismic signals, which pose severe destruction to properties and lives. Seismology concentrate on the prediction of the seismic waves to mitigate the risk.

1.2. Seismic Detector

Surface waves are captured using seismic sensors along earth’s surface. Geophone is a device which acts as a sensor for identifying ground velocities and convert them into corresponding voltages. Geophone obtains electromotive force from the vertical component of the ground motion. Geophones are light weighted, strong and requires no input power for their operation [2]. Geophone consists of magnet, which is surrounded by coils. Geophone operates at frequencies between 5 to 50 Hz. Geophone uses the principle of induction. Orientation and location of the geophone must be held properly so that no error occurs in the signal detection. The voltage generated by geophone is given as,

\[ V = n \frac{d\phi}{dx} \nu \]  

(1)

where, \( n \) is the number of coil turns, \( \frac{d\phi}{dx} \) is the rate of change of coil flux with relation to the ground motion and \( \nu \) is the vertical component of the ground motion.

Precursors (things that happen before an earthquake) such as changes in sea level, foreshocks, seismic wave velocities changes, ground deformation and so on provide the basis for predicting magnitude, place and time of the seismic event [3]. Moreover for seismic event monitoring/predicting, geo-gases such as radon (preferable), carbon dioxide, and helium are continuously monitored in some seismic prone zones. There will be
correlation between amount of radon available in region and seismic wave occurrence [4]. Observing abnormal animal behavior also provide the basis for predicting earthquake as they respond to the geophysical stimuli (caused by geophysical precursors that occur before earthquake) [5]. The data obtained from the seismic detector is undergone through the Hilbert-Huang transform to obtain instantaneous amplitudes and frequencies, useful for predicting earthquakes [6]. Thus, seismic event predictions can be done by observing animal behavior, random emissions and by the analysis of waves.

In this article, signal processing techniques are utilized for predicting a quake. For seismic wave detection, power spectral density is calculated for the signal extracted from the seismic detector. Periodogram method is used for obtaining power spectrum.

2. Mathematical Modeling

The seismic wave that is to be analyzed is stochastic in nature. To assess such signal, statistical view point is adopted. Auto-correlation is the best statistical average, which evaluates the seismic signal in time domain. Fourier transform of the auto-correlation yields power spectral density and evaluates the signal in frequency domain. The difficulties involved in determining the spectral density is that the input data lasts for only short periods and some noise will also be added to the seismic data [7]. Power spectral density can be estimated using either parametric (non-classical) or nonparametric (classical) methods. The auto-correlation sequence \( r_{yy}(l) \) is zero for all \( l >= N \) and data is periodic are the two assumptions made in nonparametric method of power estimation. Parametric methods do not have such assumptions.

Input data is finite in case of nonparametric method. Nonparametric method of power estimation begins by first determining the autocorrelation of the input data and then, finding its Fourier transform.
Periodogram is one of the nonparametric methods used to estimate the power spectrum. It was first proposed by Schuster in 1898 [8].

Let us consider a finite input sequence,

\[ y_N(n) = \begin{cases} y(n) & ; 0 \leq n < N \\ 0 & ; otherwise \end{cases} \] (2)

Here \( y_N(n) \) is defined as the product of a rectangular window and the input data. The window function is multiplied so that the data up to a certain interval can be limited.

\[ y_N(n) = w_{REC}(n).y(n) \] (3)

Auto-correlation of the input data yields

\[
\hat{r}_{yy}(l) = \frac{1}{N} \sum_{n=-\infty}^{\infty} y_N(n + l) \hat{y}_N(n)
\]

\[
= \frac{1}{N} y_N(k) * \hat{y}_N(-k) \] (4)

By applying Fourier transform to this auto-correlated sequence \( \hat{r}_{yy}(l) \), power spectral density is obtained.

\[
\widehat{p}_{PER}(e^{j\omega}) = \frac{1}{N} Y_N(e^{j\omega}) \hat{Y}_N(e^{j\omega})
\]

\[
= \frac{1}{N} |Y_N e^{j\omega}|^2 \] (5)

Here, \( Y_N e^{j\omega} \) is the DTFT (Discrete Time Fourier Transform) of \( y(n) \)

\( y(n) \) denotes \( N \)-point data

\[
Y_N(e^{j\omega}) = \sum_{n=-\infty}^{\infty} Y_N(n) e^{-jn\omega} = \sum_{n=0}^{N-1} y(n) e^{-jn\omega} \] (6)

Let the \( h_k(n) \) be the filter

\[
h_k(n) = \frac{1}{N} e^{jn\omega} w_{REC}(n)
\]

\[
= \begin{cases} \frac{1}{N} e^{jn\omega} & ; 0 \leq n < N \\ 0 & ; otherwise \end{cases} \] (7)

\( H_K(e^{j\omega}) \) is the filter frequency response (8)
The output of the filter is given as,

\[ Q_k(n) = y(n) \ast h_k(n) \]

\[ = \sum_{p=n-N+1}^{n} y(p)h_k(n-p) \]

\[ = \frac{1}{N} \sum_{p=n-N+1}^{n} y(k)e^{i(n-p)w_i} \]

Since, \( |H_k(e^{jw})|_{w=w_i} = 1 \), the spectrums of \( y(n) \) and \( q(n) \) are same at frequency \( w_i \)

\[ p_y(e^{jw_i}) = p_q(e^{jw_i}) \]

3. Simulation and Results

Step 1: Before directly applying periodogram method to the seismic data, a synthetic signal is taken and periodogram technique is applied to it. This will determine whether the method gives appropriate results. The synthetic signal is having normalized frequencies of 0.2, 0.3 and 0.5 as shown in Fig.1.

Step 2: The power spectral density of the synthetic signal is estimated using periodogram technique. From Fig.2, it is observed that maximum peaks appears at 0.2, 0.3 and 0.5 normalized frequencies. Thus periodogram method can be used for the analysis of seismic wave.

Step 3: The input signal shown in Fig.3, is obtained from Matlab file, Book_Seismic_Data.mat [9]. The source of seismic signal is placed at 100 feet under the earth and the signal is received by using geophone. These man made seismic waves are ideal to the natural ones. The length of the data is 128 and FFT length is 1024.
Step 4: The raw seismic signal that is attained is having some mean. In Fig. 4, raw seismic signal without having mean i.e., de-trended signal is shown.

Step 5: For the de-trended seismic signal, power spectral density is calculated using periodogram technique and spectrum signal is shown in Fig. 5. To obtain desired results, FIR filter is used.

Step 6: For effective results, FIR band pass filter is used, having its frequency range varying from 15 to 60 Hz. FIR (finite impulse response) filter order is taken as 9.

Step 7: Seismic signal is given as input to the FIR band pass filter and the filtered signal is plotted as Fig. 7.

Step 8: Fourier transform of the filtered seismic signal is done to obtain power spectrum. FIR filter frequencies are chosen from 15 to 60 Hz as earthquakes will be having these frequencies.

From Fig. 8, the maximum peak is observed at 0.1239 normalized frequency,

\[ w = \frac{2\pi f}{f_s} = 0.1239\pi \]

\[ f_s = 500 \]

\[ \frac{2\pi f}{500} = 0.1239\pi \]

\[ f = 30.975 \text{ Hz} \]

Where, \( f_s \) is the sampling frequency

\( f \) is the frequency at which maximum power is observed

Fig. 1. Synthetic Signal.  
Fig. 2. PSD for synthetic signal.
Fig. 3. Raw Seismic signal.

Fig. 4. Detrended seismic signal.

Fig. 5. Raw signal spectrum.

Fig. 6. FIR band pass filter.

Fig. 7. FIR band pass filtered signal.

Fig. 8. PSD of seismic signal.
4. Conclusion

In this research article, prediction of quake occurrence is estimated using signal processing technique. Periodogram is the basic signal processing technique used. Seismic parameters such as energy, magnitude, wavelength, and frequency of the quake are assessed from the spectrum signal. By observing these parameters, we can anticipate the quake occurrence.

5. References
