Abstract

The analysis of circular plates is very important in real life application for different type of loading conditions. For simple loading condition like central loading, the governing equation are available in polar coordinates which can be easily differentiated and can be converted into the Cartesian coordinate for finding the exact solution for the deformation field. But when loading condition changes from centre position to any other position within the domain the governing equation changes from simple polar coordinate to in the summation series expansion which is very deadening process and hence becomes difficult to differentiate for getting the exact solution Finite Element Analysis is Numerical technique for getting the approximate solution for the case of complex loading condition as well as for the case which do not have exact solution and hence ANSYS is opted for the analysis of simply supported asymmetrically loaded circular plate. Circular plate is loaded on the diameter other than the centre and for validating the numerical analysis, convergence test is applied by discretising the circular plate into more number of elements. The circular plate is analysed for the three cases in Ansys and validated by convergence test and one of the case is also verified by optical experiment and compared with ansys result.

AMS Subject Classification: 65N30, 65L60, 65R99
1 Introduction

Plates are extensively used in many real-life applications, for example, in water tanks, turbine disks, roofs, and floors of buildings, etc. Plates used in such applications are subjected to lateral loads because of bending of plates. Bending of plates represents deflection under different loading conditions, which sometimes causes the failure of plates and hence analysis of such loading is very important in real life for preventing failure. For standard structures like centrally loaded simply supported and clamped circular plates, the true solution is available for finding the displacement, slope, and curvature values for analysis but for complex loading conditions, getting exact solutions is a tedious process and hence leads to an alternative method like Finite Element Analysis (FEA) (ANSYS) which is a numerical technique for finding the approximate solutions. The objective of this work is to determine the deformation field for non-centric and centrally loaded simply supported circular plates using numerical and analytical methods, respectively, and validate it by convergence tests as well as experiments.

2 Mechanics Of Circular Plates

If the load acting on a circular plate through its center (see [4]), the deflection is symmetrically distributed about the axis perpendicular to the plate.

The governing equation of a circular plate when load is acting at a center is:

$$\frac{1}{r} \frac{d}{dr} \left\{ r \frac{d}{dr} \left[ \frac{1}{r} d \frac{d \omega}{dr} \right] \right\} = \frac{q}{D} (1)$$

The equation can be easily differentiated for obtaining the deflection equation in the term of radius of circle which can be again converted into Cartesian coordinate by putting the relation $x^2 + y^2 = r^2$.

Deflection and slope equation respectively for centrally loaded simply supported circular plate in polar coordinates.

$$\omega = \frac{P}{16 \pi D} \left[ \frac{3 + \mu}{1 + \mu} (a^2 - r^2) + 2r^2 \log \frac{r}{a} \right] (2)$$

$$\frac{d \omega}{dr} = \frac{P}{4 \pi D} \left[ r \log \frac{r}{a} - \frac{r}{1 + r} \right] (3)$$
For finding the maximum deflection and slope at the center substitute $r=0$ in the equation number (2) and (3). we get

$$\frac{d\omega}{dr} = 0$$  (4)

$$\omega_{\text{max}} = \frac{3 + \mu}{1 + \mu} \left( \frac{Pa^2}{16\pi D} \right)$$  (5)

Equation of circle is $x^2 + y^2 = r^2$ with center $(0, 0)$ (i.e.) $(x, y)$ Substitute the circle equation in (2), the modified deflection equation is

$$\omega = \frac{P}{16\pi D} \left[ 2x^2 \log \left( \frac{x^2 + y^2}{a^2} \right) + 2y^2 \log \left( \frac{x^2 + y^2}{a^2} \right) + 3 + \mu \left( a^2 - x^2 - y^2 \right) \right]$$  (6)

By using this modified deflection equation, slope and curvature equation is obtained by differentiating partially with respect to $x$ and $y$. The equation (7) and (8) is the slope equation in the $x$ and $y$ direction respectively.

$$\frac{\partial\omega}{\partial x} = \frac{P}{16\pi D} \left[ 2x \log \left( \frac{x^2 + y^2}{a^2} \right) - \frac{4x}{1 + \mu} \right]$$  (7)

$$\frac{\partial\omega}{\partial y} = \frac{P}{16\pi D} \left[ 2y \log \left( \frac{x^2 + y^2}{a^2} \right) - \frac{4y}{1 + \mu} \right]$$  (8)

The equation (9) is the governing equation of circular plate when load is acting other than centre (see [1])

$$D \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right] \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \omega_0}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \omega_0}{\partial \theta^2} \right] + k \omega_0 = q(r, \theta)$$  (9)

for static bending of an isotropic plate, not resting on an elastic foundation and without thermal loads, equation (9) becomes (10) the solution content $\omega_0$ to this solution consist of two parts: a homogeneous solution $\omega_h$ such that

$$D \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right] \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \omega_h}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \omega_h}{\partial \theta^2} \right] = 0$$  (10)

and a particular solution $\omega_p$ so that

$$\omega_0 = \omega_h + \omega_p$$  (11)

The homogeneous solution may be expressed in the following form

$$\omega_h(r, \theta) = \sum_{n=0}^{\infty} a_n(r) \cos n\theta + \sum_{n=1}^{\infty} b_n(r) \sin n\theta \quad (12)$$
where $a_n$ and $b_n$ are functions of $r$ only. Substitution of equation (12) into equation (11) leads to the following set of equation for $a_n$ and $b_n$

$$a_0 = A_0 + B_0 r^2 + C_0 \log r + D_0 r^2 \log r$$ (13)

$$a_1 = A_1 r + B_1 r^3 + C_1 r^{(-1)} + D_1 r \log r$$ (14)

$$b_1 = E_1 r + F_1 r^3 + G_1 r^{(-1)} + H_1 r \log r$$ (15)

$$a_n = A_n r^n + B_n r^{-n} + C_n r^{n+2} + D_n r^{-n+2}$$ (16)

$$b_n = E_n r^n + F_n r^{-n} + G_n r^{n+2} + H_n r^{-n+2}$$ (17)

for $n = 2, 3, \ldots$ where $A_n, \ldots, H_n$ are constants that are determined using boundary condition. Since the equation (12) is in sine and cosine series expansion which is deadening process to get slope and curvature values and hence the finite element analysis which is also called numerical analysis is opted for analyzing this type of loading condition.

### 3 Modelling Of Simply Supported Circular Plate

The simply supported circular plate of 20mm radius is modelled in Ansys by using the element 4 node SHELL181. The youngs modulus and poisson ratio for the modelled circular plate is taken as 2500 N/mm$^2$ (Perspex sheet) and 0.3 respectively. All translational exterior node and one rotational degree of freedom is taken as zero for giving the boundary condition. A known displacement is applied by creating the hard point for all three cases which is shown in table (1). The obtained contours for all three cases of circular plate after convergence test is shown in figure 1. Where first one (a) is for the case 1 and (b),(c) are the case 2 and case 3 respectively.

**Table 1: Loading condition for all three cases of simply supported circular plate, where radius of circle is $r=20$mm**

<table>
<thead>
<tr>
<th>Case 1</th>
<th>At centre $(x=0, y=0), 0.01$mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 2</td>
<td>At $(x=r/2, y=0), 0.01$mm</td>
</tr>
<tr>
<td>Case 3</td>
<td>At $(0.02$mm at $x=-r/4, y=10$ and $0.01$mm at $x=r/2, y=0$),</td>
</tr>
</tbody>
</table>
4 Matlab Simulation And Experiment Validation

In Ansys, there is no direct way to obtain the curvature contours. So all the nodal slope values for all three cases were imported into MATLAB and run through the algorithm which differentiate the imported slope values for getting the curvature contours. Since the data obtained in the ansys is not distributed in their relative position that is data was scattered so The obtained slope values along with the coordinates for the entire plate along X and Y direction are imported to matlab for obtaining the slope, curvature contour and slope graph along the diameter for all three cases of loading. Scatteredinterpolant command is used for fitting three dimensional functions for the data imported in the MATLAB simulation and result obtained for the case 3 through matlab simulation is shown in figure 3 and figure 4 and also the graph along diameter for all three cases is shown in figure 2.

Optical experiments (see [2]) comes under the experimental mechanics where the images of reflective plates captured in two states are used for finding the deformation field. obtained two images were captured one before giving the displacement and second one after giving the displacent and the case 1 is
varified using IIT (see [3]), ( iterated intensity integration technique) algorithm.

5 Result

For centrally loaded simply supported circular plate two solution is available one true solution and second numerical analysis but for the asymmetrical condition numerical analysis is the only alternative available and hence Convergence method is applied for validating the numerical analysis and case 1 is verified through optical experiment and an error of 8.803 % is obtained when compared.
6 Acknowledgement

I express my deep and sincere gratitude to Dr. G. Subramanian, visiting professor for his sustained support, inspiration and guidance he has given. I would also like to extend my gratitude to the faculty and lab assistants for their incessant support in technical assistance in making this project a success.

References


