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Neighbourhood prime labelling on some path related graphs

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Abstract

The neighbourhood prime labelling of a graph G is defined as a function $f : V(G) \longrightarrow \{1, 2, 3, ..., n\}$ which is bijective and if for every vertex of G with degree greater than 1, gcd $\{f(u) : u \in N(v)\} = 1$. A graph is called neighbourhood prime if it admits neighbourhood prime labelling. In this paper we introduce m-corona and prove that the corona products $P_n \odot P_n$, the shadow graph of paths P_n, m-corona of paths P_n are all neighbourhood prime graphs.

AMS Subject Classification: 05C78

Key Words and Phrases: Labelling, Prime labelling, paths, neighbourhood prime labelling, corona product, shadow graphs, m-corona graphs, 2-corona graphs.

1 Introduction

Graph labelling act as a milestone in path of brightening the practical world of maths. The concept of graph labelling was originated

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from the the paper[2] by Alex Rosa in the year 1967. Later on, different classes of graph labelling was introduced by many authors. Graph labelling is a process to assign the labels, traditionally represented by integers to the elements of graph. For a graph G, a vertex labelling is a function from the set of vertices of G to set of labels. A graph with such a function is called a vertex labelled graph. Graph labelling plays an important role in various fields like communication networks, coding theory, radar, astronomy, crystallography, circuit design etc.

Entringer introduced the concept of prime labelling and was later studied by Tout et al, [10]. Consider the graph G with $V(G)$ as the set of vertices and $E(G)$ as the set of edges. The set of all adjacent vertices of a vertex v in G is called the neighbourhood of the vertex v which is denoted as $N(v)$. For the graph G of n vertices, a bijective function f from the vertex set $V(G)$ to the set $\{1, 2, 3, ..., n\}$ is a prime labelling if for every edge uv, $gcd\{f(u), f(v)\} = 1$, where $uv \in E(G)$. A graph is said to be prime if it admits prime labelling. A prime labelling has many applications in chemistry, relational databases, graph isomorphism problems etc.

Prime labelling of graphs obtained by switching the vertices of P_n, K_n and wheel graph W_n are discussed in [6]. Vaidya and Kanani proved that the graphs obtained by fusion, duplication, and vertex switching in C_n is a prime graph, [12]. The prime labelling of ladder graph and coprime labelling of complete bipartite graphs are verified in [1].

Neighbourhood prime labelling was originated by S. K Patel and N. P Shrimali in the year 2015, [7]. The neighbourhood prime labelling of a graph G is defined as a function $f : V(G) \longrightarrow \{1, 2, 3, ..., n\}$ which is bijective and if for every vertex of G with degree greater than 1, gcd $\{f(u) : u \in N(v)\} = 1$. A graph is called neighbourhood prime if it admits neighbourhood prime labelling.

There were studies on neighbourhood prime labelling of graphs obtained by union of 2 cycles, 2 wheels and finite number of paths in [8]. Also neighbourhood prime labelling of graphs like ladder, gear, friendship graphs, triangular book, coconut tree are discussed in

[3]. Neighbourhood prime labelling of graphs obtained from crown graphs and 2-corona $G \odot S_2$ where G is an H-graph are proved in the paper [5] by Rajesh Kumar T.J and Mathew Varkey T.K. Motivated by their studies here we introduce m-corona and we prove that the corona products $P_n \odot P_n$, the shadow graph of paths P_n, m-corona of paths P_n are all neighbourhood prime graph.

2 Corona Products

Definition 1. For graphs G and H, corona product $G \odot H$ is obtained by taking one copy of graph G and $\mid V(G) \mid$ copies of graph H and joining each vertex of i^{th} copy of graph H to the i^{th} vertex of the graph G, where $1 \leq i \leq \mid V(G) \mid$, [9].

Definition 2. $P[t, v]$ is defined as the set of all prime numbers p such that $t < p \leq v$, for any numbers t and v.

Theorem 3. *The corona product $P_n \odot P_n$ is a neighbourhood prime graph, for any n.*

Proof. Let G be the corona product $P_n \odot P_n$ of paths P_n. As in the figure 1 name the path to which n copies of P_n are attached as P_n^* and all the n copies of P_n attached to P_n^* as $P_{n_i}$where $i = 1, 2, ..., n$.
Here $\mid V(G) \mid= n^2 + n$. Now denote the successive vertices of path P_n^* as $v_1, v_2, ..., v_n$ and each vertex of P_{n_i} connected to v_i as $v_{i1}, v_{i2}, ..., v_{i(n-1)}, v_{in}$, where $i = 1, 2, ...n$.

When $n = 1$ the result is trivial and for $n \geq 2$, we define labelling $f : V(G) \longrightarrow \{1, 2, ..., n(n + 1)\}$;
By appyling Prime Number Theorem [4], there will be atleast n prime numbers between 1 and $n(n + 1)$. The vertices of path P_n^* is labelled with highest possible prime numbers less than or equal to $n(n + 1)$ and all other vertices of G with remaining integers of the set $1, 2, ..., n(n + 1)$. There are 2 cases;
Case I
Consider vertices of P_n^* if $f(v_i) = P[\frac{n(n+1)}{2}), n(n + 1)]$:
Since $f(v_{i+1}), f(v_{i-1})$ are prime numbers;

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$gcd\{f(v_{i1}), ..., f(v_{in}), f(v_{i+1}), f(v_{i-1}) : f(v_{i1}), f(v_{i2}), ..., f(v_{in}), f(v_{i+1}), f(v_{i-1}) \in N(v_i)\} = 1$, where $1 \leq i \leq n$.
Also since $f(v_i)$ is a prime number,
$gcd\{f(v_{i(j-1)}), f(v_{i(j+1)}), f(v_i) : f(v_{i(j-1)}), f(v_{i(j+1)}), f(v_i) \in N(v_{ij})\}$
$= 1$, where $1 \leq i \leq n$, $1 < j < n$.

Case II
Consider vertices of P_n^* if $f(v_i) \in P[1, \frac{n(n+1)}{2}]$:
The labelling must satisfy :
$f(v_{i2}) \neq cf(v_i)$ and $f(v_{i(n-1)}) \neq cf(v_i)$, where $1 \leq i \leq n$ and positive integer c.
Since $f(v_i)$ is a prime number,
$gcd\{f(v_{i2}), f(v_i) : v_{i2}, v_i \in N(v_{i1})\} = 1$, and
$gcd\{f(v_{i(n-1)}), f(v_i) : v_{i(n-1)}, v_i \in N(v_{in})\} = 1$
Hence the graph obtained by the corona product $P_n \odot P_n$ of paths is a neighbourhood prime graph. □

Illustration

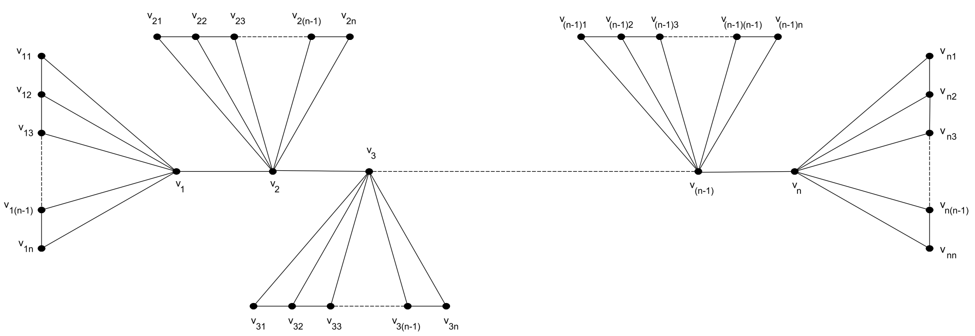

Figure 1: $P_n \odot P_n$

3 Shadow Graphs

Definition 4. The shadow graphs of any connected graph G is obtained by taking 2 copies of G say G' and G'' and joining each vertex v' in G' to the neighbours of the corresponding vertex v'' in

G'' [11].

Theorem 5. *The shadow graph of paths P_n is a neighbourhood prime graph, for any n.*

Proof. Consider G as the shadow graph of the path P_n. Here $\mid V(G) \mid = 2n$.
As in the figure 2, name the two paths with n vertices as P'_n and P''_n. Let $v_1, v_2, ..., v_{n-1}, v_n$ be consecutive vertices of the path P'_n and $u_1, u_2, ..., u_{n-1}, u_n$ be consecutive vertices of the path P''_n.
Define labelling $f : V(G) \longrightarrow \{1, 2, ..., 2n\}$;
When $n = 1$ the result is trivial and for $n \geq 2$ there are 2 cases:
CASE I : For path P'_n:
$f(v_i) = 2i$, where $1 \leq i \leq n$.

CASE II : For path P''_n:
$f(u_i) = 2i - 1$, where $1 \leq i \leq n$.
Here we note that;
$i.$) For $1 < i < n$:
Since $f(v_{i+1})$ and $f(u_{i+1})$, $f(v_{i-1})$ and $f(u_{i-1})$ are pairs of integers which are consecutive.
$gcd\{f(v_{i+1}), f(u_{i+1}), f(v_{i-1}), f(u_{i-1})\} = 1$,
where $v_{i+1}, u_{i+1}, v_{i-1}, u_{i-1}$ are the neighbourhoods of u_i as well as v_i.
ii) For the $i = 1, n$:
Since $f(u_2), f(v_2)$ are integers which are consecutive.
$gcd\{f(u_2), f(v_2)\} = 1$, where u_2, v_2 are neighbourhoods of u_1 as well as v_1.
Since $f(u_{n-1}), f(v_{n-1})$ are integers which are consecutive.
$gcd\{f(u_{n-1}), f(v_{n-1})\} = 1$, where u_{n-1}, v_{n-1} are neighbourhoods of v_n as well as u_n. Hence we proved that shadow graph of path P_n is a neighbourhood prime graph. □

Illustration

4 m-Corona Graphs

Definition 6. m-Corona of any graph G with n vertices is obtained by identifying the center vertex of the star graph $K_{1,m}$ at

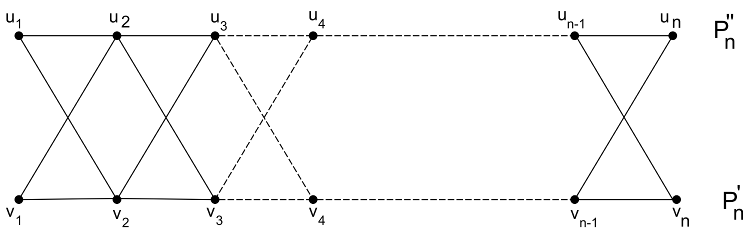

Figure 2: Shadow graph

each of the n vertices of graph G.

Theorem 7. *The m-corona of paths P_n is a neighbourhood prime graphs, for any n and m.*

Proof. Let G be graph obtained from the m-corona of paths P_n. Note that $\mid V(G) \mid= n(m+1)$. Name the path in G with n vertices as P_n^* and the successive vertices of P_n^* as $v_1, v_2, ..., v_n$. Let each of pendant vertices v_{i1}, v_{i2},....,v_{ij}, $j = 1, 2, ..., m$ be adjacent to corresponding vertices v_i in P_n, where $i = 1, 2, ..., n$.

To define the labelling $f : V(G) \longrightarrow \{1, 2, ..., n(m+1)\}$;
When $n = 1$, the result is trivial.
For $n \geq 2$, there are 2 cases.
CASE I: For the vertex v_i:
$f(v_i) = mi$, where $i = 1, 2, ..., n$.

CASE II: For the vertices v_{ij}:
$f(v_{ij}) = mi - j$, where $i = 1, 2, ..., n$, $j = 1, 2, ..., m$.

Here we note that, since $f(v_{i1})$, $f(v_{i2})$,...,$f(v_{ij})$ are consecutive integers, $gcd\{f(v_{i1}), f(v_{i2}), ..., f(v_{ij})\} = 1$ and also the neighbourhood of v_{i1},v_{i2},...,v_{ij} is the single vertex v_i.
We have,
i.) $gcd\{f(v_{i+1}), f(v_{i-1}), f(v_{i1}), .., f(v_{ij}) : v_{i+1}, v_{i-1}, v_{i1}, .., v_{ij} \in N(v_i)\} = 1$, where $1 < i < n$, $j = 1, 2, ..., n$.
ii.) $gcd\{f(v_{11}), .., f(v_{1m}), f(v_2) : v_{11}, .., v_{1m}, v_2 \in N(v_1)\} = 1$.
iii.)$gcd\{f(v_{n1}), .., f(v_{nm}), f(v_{n-1}) : v_{n1}, .., v_{nm}, v_{n-1} \in N(v_n)\} = 1$.

Hence we proved that the graph obtained by the m-corona of paths P_n is a neighbourhood prime graph.

□

We have a deduction, which appeared in [5].

Definition 8. 2-*Corona* of any graph G of n vertices is obtained by joining the center vertex of the star graph $K_{1,2}$ at each of the n vertices of graph G.

Corollary 9. *The 2-corona of path P_n is a neighbourhood prime graph, for any n.*

Illustration

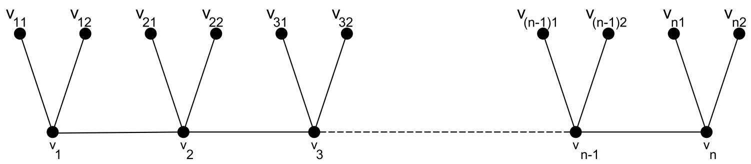

Figure 3: 2-corona of path P_n

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