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State dependent admission of demands in a finite storage system

*Diana Tom Varghese, †Dhanya Shajin
Department of Mathematics
Amrita School of Arts and Sciences, Kochi-682024
Amrita Vishwa Vidyapeetham, India.

Abstract

We consider an $M/M/1/S$ inventory model under (s, S) policy. Arrival follows a non-homogeneous Poisson process. Exponentially distributed service time and lead time. We assume that a new arrival joins the queue only if the inventory is not less the number of demands in the system. Using matrix analytic method, the system characteristics are evaluated. A cost function is constructed to numerically investigate the optimal pair (s, S).

AMS Subject Classifications: 60J10, 60K25, 90B05, 90B22.

Keyword

Queueing-inventory, non-homogeneous Poisson process.

1 Introduction

Queueing inventory has its origin in Melikov et al. [4] and Sigman & Levy [6]. Schwarz et al. [5] discussed $M/M/1$ model with a storage system having exponentially distributed lead time. They considered (s, Q) and (s, S) policies and derived PFS. In a Markovian environment Gaver et al. [2] discussed an efficient computational approach to the analysis of finite birth-and-death models. For further details about various inventory models with positive service time we refer to a survey paper by Krishnamoorthy et al. [3].

The remaining part of the article is arranged as follows. In Section 2, the model under study is described. System state distribution is derived in Section 3. Some important system performance measures, numerical examples and cost analysis are discussed in Section 4.

2 Model description

Consider an $M/M/1/S$ model with a finite storage system. Arrival of demands follow a non-homogeneous Poisson process with rate Λ_j which is dependent on the on-hand

*E-mail:myselfdianatom@gmail.com
†Author for correspondence: E-mail:dhanya.shajin@gmail.com

inventory level j, $0 \leq j \leq S$ and $\Lambda_0 \leq \Lambda_1 \leq ... \leq \Lambda_S$. Exponentially distributed service and lead time with parameter ν and η respectively. An (s, S) ordering policy is adopted.

Assumption

If $n < i$, then a new arrival joins the system where n, i represent the number of customers in the system and that of items in the inventory. Otherwise, the new arrival departs.

3 Steady state analysis

Let $N(t)$ and $I(t)$ denote, respectively the number of customers in the system, the number of items in the inventory at time t. Then by the above assumption we have $\Omega = \{(N(t), I(t)), t \geq 0\}$ is a continuous time Markov chain which is a level dependent quasi-birth and death process with state space $\{(m, i), 0 \leq m \leq S, m \leq i \leq S\}$. Thus the infinitesimal generator matrix $\mathcal{W}$ is of the form

$$\mathcal{W} = \begin{pmatrix} Z_1^{(0)} & Z_0(0) & & & & \\ Z_2^{(1)} & Z_1^{(1)} & Z_0^{(1)} & & & \\ & Z_2^{(2)} & Z_1^{(2)} & Z_0^{(2)} & & \\ & & \ddots & \ddots & \ddots & \\ & & & Z_2^{(S-1)} & Z_1^{(S-1)} & Z_0^{(S-1)} \\ & & & & Z_2^{(S)} & Z_1^{(S)} \end{pmatrix}. \tag{1}$$

The transition rates are:

(a) Transitions due to arrival:

$$(m, i) \to (m+1, i): \text{ rate } \Lambda_i \text{ for } 0 \leq m \leq S-1, m+1 \leq i \leq S.$$

(b) Transitions due to service completions:

$$(m, i) \to (m-1, i-1): \text{ rate } \nu \text{ for } 1 \leq m \leq S, m \leq i \leq S.$$

(c) Transitions due to replenishment:

$$(m, i) \to (m, S): \text{ rate } \eta \text{ for } 0 \leq m \leq s, m \leq i \leq s.$$

Entries of $Z_0^{(n)}, 0 \leq n \leq S-1$ are given in (a); that of $Z_2^{(n)}, 1 \leq n \leq S$ are given in (b) and those in $Z_1^{(n)}, 0 \leq n \leq S$ correspond to transition rates given by (c). In addition diagonal entries in $Z_1^{(n)}, 0 \leq n \leq S$ are non-positive. All other transitions have rate zero.

Let $\boldsymbol{\pi}$ be the stationary probability vector of the generator $\mathcal{W}$. Then

$$\boldsymbol{\pi}\mathcal{W} = \mathbf{0} \text{ and } \boldsymbol{\pi}\mathbf{e} = 1 \tag{2}$$

where $\boldsymbol{\pi} = (\boldsymbol{\pi}_0, \boldsymbol{\pi}_1, ..., \boldsymbol{\pi}_S)$ with $\boldsymbol{\pi}_n = (\pi(n,n), \pi(n,n+1), ..., \pi(n,S))$ for $0 \leq n \leq S$.

The first equation in (2) implies

$$\begin{aligned} \boldsymbol{\pi}_0 Z_1^{(0)} + \boldsymbol{\pi}_1 Z_2^{(1)} &= \mathbf{0}, \\ \boldsymbol{\pi}_{n-1} Z_0^{(n-1)} + \boldsymbol{\pi}_n Z_1^{(n)} + \boldsymbol{\pi}_{n+1} Z_2^{(n+1)} &= \mathbf{0}, \quad 1 \leq n \leq S-1 \\ \boldsymbol{\pi}_{S-1} Z_0^{(S-1)} + \boldsymbol{\pi}_S Z_1^{(S)} &= \mathbf{0}. \end{aligned}$$

Solving the above set of equations we have

$$\boldsymbol{\pi}_n = \boldsymbol{\pi}_{n-1} B_n, 1 \leq n \leq S \tag{3}$$

$$\text{where } B_n = \begin{cases} -Z_0^{(S-1)} \left(Z_1^{(S)} \right)^{-1}, & n = S \\ -Z_0^{(n-1)} \left(Z_1^{(n)} + B_{n+1} Z_2^{(n+1)} \right)^{-1}, & 1 \leq n \leq S-1. \end{cases}$$

Now using $\boldsymbol{\pi}\mathbf{e} = 1$, we get

$$\boldsymbol{\pi}_0 \left[I + \sum_{n=1}^{S} \prod_{i=1}^{n} B_i \right] \mathbf{e} = 1. \tag{4}$$

4 System characteristics

- Mean number of customers in the system: $E_C = \sum_{n=1}^{S} \sum_{i=n}^{S} n\pi(n,i)$.
- Expected number of items in the inventory: $E_I = \sum_{i=1}^{S} i\pi(0,i) + \sum_{n=1}^{S} \sum_{i=n}^{S} i\pi(n,i)$.
- Expected re-order rate: $E_R = \nu \sum_{n=0}^{s+1} \pi(n, s+1)$.
- Expected loss rate of customers: $E_L = \sum_{n=0}^{S} \sum_{i=0}^{n} \Lambda_i \pi(n,i)$.
- Expected rate of replenishment: $E_{RR} = \eta \sum_{n=0}^{s} \sum_{i=n}^{s} \pi(n,i)$.

Numerical examples

Next we proceed to a few numerical examples. Here demands occur according to a non-homogeneous Poisson process with rate Λ_j which depends on the on-hand inventory level $j; 0 \leq j \leq S$ and $\Lambda_0 \leq \Lambda_1 \leq ... \leq \Lambda_S$. We use

$$\Lambda_j = \Lambda j^{\gamma}, 0 \leq j \leq S \tag{5}$$

where $0 \leq \gamma \leq 1$ (see Alfares [1]). It is an increasing function of the on-hand inventory level. If $\gamma = 0$, then the demand rate is homogeneous Poisson process with rate Λ.

From Tables 1, 2, 3 we observe the system behaviour with respect to certain parameters.

Λ	E_C	E_I	E_R	E_L
3	1.3531	20.3000	0.2022	0.0039
3.5	1.9842	20.1178	0.2312	0.0170
4	2.9148	19.9415	0.2577	0.0615
4.5	4.1894	19.7834	0.2803	0.1791
5	5.7077	19.6583	0.2974	0.4164
5.5	7.2493	19.5719	0.3087	0.7903

Table 1: Effect of Λ for $(S, s, \nu, \gamma, \eta) = (30, 12, 7, 0.1, 2)$

ν	E_C	E_I	E_R	E_L
6	6.2420	24.4863	0.2459	0.1373
6.5	4.5214	24.4642	0.2493	0.0578
7	3.3908	24.4507	0.2508	0.0245
7.5	2.6541	24.4412	0.2514	0.0110
8	2.1592	24.4334	0.2517	0.0053
8.5	1.8122	24.4267	0.2519	0.0028

Table 2: Effect of ν for $(S, s, \Lambda, \gamma, \eta) = (35, 15, 4, 0.1, 3)$

η	E_C	E_I	E_R	E_L
0.5	5.4366	22.0672	0.1805	0.5063
1	7.3208	25.5976	0.2263	0.4371
1.5	8.3758	26.9155	0.2480	0.3860
2	9.0306	27.5839	0.2604	0.3572
2.5	9.4625	27.9828	0.2685	0.3409
3	9.7618	28.2463	0.2740	0.3310

Table 3: Effect of η for $(S, s, \Lambda, \nu, \gamma) = (40, 18, 5, 7, 0.1)$

Cost analysis

Now we introduce the cost function $K(s, S)$ as $K(s, S) = [D_0 + (S - s)D_1]E_R + D_2 E_I + D_3 E_C + D_4 E_L$
where D_0: fixed cost for placing an order,
D_1: procurement cost / unit item,
D_2: holding cost of inventory / unit / unit time,
D_3: holding cost of customers / unit / unit time,
D_4: cost due to the loss of customers / unit / unit time.

We fix the values of the parameters: $\Lambda = 3, \nu = 5, \eta = 1.5, \gamma = 0.1, D_0 = \$100, D_1 = \$10, D_2 = \$1, D_3 = \$0.5, D_4 = \$20$ and vary (s, S).

From Table 4 we obtain $(s, S) = (1, 18)$ and corresponding cost is \$63.4663. Also we obtain the cost function is convex in nature (see Figure 1).

(s,S) pair	(1,18)	(2,19)	(3,20)	(4,22)	(5,23)	(6,24)	(7,25)
cost in$	63.4663	64.6943	65.8870	67.0687	68.2591	69.4640	70.6809

Table 4: Optimal (s,S) pair and corresponding minimum cost

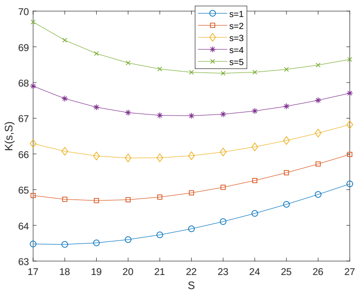

Figure 1: S verses $K(s,S)$

Conclusion

We studied $M/M/1/S$-inventory model to which arrival follows a non-homogeneous Poisson process. Exponentially distributed lead time. We assume that a new arrival joins the queue only if $I(t) \geq N(t)$. Cost function was defined for minimize loss. In a future paper we proposed the present work to generally distributed service process and lead time.

References

[1] H. K. Alfares, *Inventory model with stock-level dependent demand rate and variable holding cost*, International Journal of Production Economics, 108, 256-265 (2007).

[2] D. P. Gaver, P. A. Jacobs, G. Latouche, *Finite Birth and Death models in Randomly changing Environments*, Advances in Applied Probability, 16(4), 715-731 (1984).

[3] A. Krishnamoorthy, B. Lakshmy, R. Manikandan, *A survey on Inventory models with positive service time*, OPPSEARCH, 48(2), 153-169 (2011).

[4] A. Z. Melikov, A. A. Molchanov, *Stock optimization in transportation/ storage systems*, Cybernetics and Systems Analysis, Vol.28, No.3, 484 - 487 (1992).

[5] M. Schwarz, C. Sauer, H. Daduna, R. Kulik, R. Szekli, *M/M/1 queueing systems with inventory*, Queueing System 54:55-78 (2006).

[6] K. Sigman, D Simchi-Levi, *Light traffic heuristic for an M/G/1 queue with limited inventory*, Annals of Operations Research, 40:371-380 (1992).