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EDGE-ODD GRACEFUL LABELING OF PATH MERGING WITH FAN AND CIRCUIT MERGING WITH NULL GRAPH

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Abstract: Kaneria et. al. [2014a] discussed graceful labeling for some star related graphs, and Kaneria et, al. [2014b] graceful labeling for open star of graphs. Gao [2007] studied odd graceful labelings of some union graphs like as path, star, and cycle. Seoud and Abdel-Aal [2013] analyzed a new class of odd graceful graphs. Vaidya and Lekha [2010a] obtained some new graphs having odd graceful labeling. Vaidya and Lekha also found new families of odd graceful graphs from wheel and star. Vaidya and Shah got graceful and odd graceful labelings for some graphs related path and circuit. In this article, the edge-odd graceful labelings of each graph Pn ∗ nF5, Pn ∗ nF7, C7 ∗ 3Fn,, Cn ∗ N2, and Cn + k ∗ N2 are obtained.

Key words: edge -odd graceful labeling, edge -odd graceful graph.

1. INTRODUCTION

Badr, Moussa & Kathiresan [2011] proved the following: (1). The cycle Cn is odd graceful if n is even (n ≥ 4); (2). The crown graph C_n Θ mK_1 is odd graceful if n is even (n ≥ 4); (3). The subdivision of ladders S(Ln) is odd graceful.

Pradhan, P., and Kamesh Kumar [2014] found the results mentioned below: (1). The graph obtained by adding r - pendant edges to each vertex of Kn in the graph $P_2 + K_n$ admits graceful labeling; (2). The graph obtained by adding r - pendant edges to each vertex of in the graph $P_2 + K_n$ admits graceful labeling; (3). The graph $(P_2 + K_n) \odot rK_1$ admits graceful labeling where $n \geq 2(r - 1)$ when $r > 1$ and $n > 0$ when $r = 1$; (4). The graph obtained by adding pendant edge to each pendant vertex of hairy cycle $C_n \odot 1K_1$, $n \equiv 3 \pmod 4$ admits graceful labeling.

2. EXISTING WORK

The following definitions are first given.

Definition 2.1; $P_n * nF_5$ is a connected graph whose vertex set is $\{v_1, v_2, \ldots, v_{6n}\}$, and edge set is $\{v_{5n+i}\, v_{5n+i+1}$: i varies from 1 to (n-1)$\} \cup \{v_{5n+i}\, v_{(i-1)5+j}$; i = 1 to n and j = 1 to 5 $\} \cup \{v_i v_{i+1}$; i varies from 1 to 5n ; $i \neq 0 \pmod 5$ }.

Definition 2.2: $P_n * nF_7$ is a connected graph whose vertex set is $\{v_1, v_2, \ldots, v_{6n}\}$, and edge set is $\{v_i v_{i+1}$; i=1 to 7n, where either $i \neq 0 \pmod 7$ or $i \neq 7n+1, 7n+2, \ldots, 8n-1\} \cup \{v_{7n+1} v_{(i-1)7+j}$; i = 1 to n ; j = 1 to 7}. Here n-copies of (F_7) are $\{v_i v_{i+1}$; i = 1 to 7n ; $i \neq 0 \pmod 7$ }, and one copy of P_n is $\{v_i v_{i+1}$; i = 7n+1, 7n+2,…,(8n-1)}.

Definition 2.3: $C_7 * 3F_n$ is a connected graph whose vertex set is Vertex set = $\{u_0, u_1, u_2, u_3, u_1', u_2', u_3', v_1, v_2, \ldots, v_{3n}\}$, and edge set is $\{u_1 v_i, u_2 v_{n+1}, u_3 v_{2n+1}$; i = 1 to n$\} \cup \{v_i v_{i+1}$, i 1 to 3n, $i \neq n, 2n, 3n\} \cup \{u_0 u_1', u_1' u_2', u_2' u_3', u_3' u_3, u_3 u_2, u_2 u_1, u_0 u_1, u_0 u_2', u_0 u_3'\}$. 3-copies of fan (Fn) are $\{u_1 v_i, u_2 v_{n+i}, u_3 v_{2n+i}$; i = 1 to n } ∪ $\{v_i v_{i+1}$, i = 1+3n, $i \neq n, 2n, 3n\}$.

Definition 2.4: $P_n + \overline{K_2}$ is a connected graph whose vertex set is $\{u, v, v_1, v_2, \ldots\ldots, v_n\}$, and edge set is $\{uv_i, vv_i$: i= 1 to n $\} \cup \{v_i v_{i+1}$: i = 1 to (n-1) }.

Definition 2.5: $C_n + \overline{K_2}$ is a connected graph whose vertex set is $\{u, v, v_1, v_2, \ldots\ldots, v_n\}$, and edge set $=\{uv_i, vv_i$: i = 1 to n $\} \cup \{v_i v_{i+1}$: i = 1 to (n-1)$\} \cup \{v_1 v_n\}$

Definition 2.6: $C_{n+k} + \overline{K_2}$ is a connected graph whose vertex set is $\{u, v, v_1, v_2, \ldots\ldots, v_n\}$, and edge set $=\{uv_i, vv_i$: i = 1 to n $\} \cup \{v_i v_{i+1}$: i = 1 to (n-1)$\} \cup \{v_{n+k} v_1\}$.

3. PROPOSED WORK

Edge-odd graceful of the above graphs:

Now the theorem is started.

Theorem 3.1: The connected graph $P_n * nF_5$ is edge-odd graceful.

Proof: One of the labelings for vertex set is as follows:

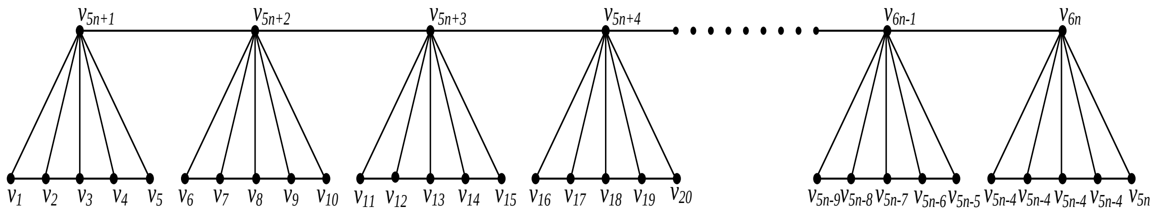

Figure.1 $P_n * nF_5$ graph

Define $f : E(G) \rightarrow \{1, 3, 5, \ldots, (2q-1)\}$ by

$f(v_i v_{i+1}) = 20k+1$, $i = 5k+1$, $i = 1$ to $5n$, $i \neq 0 \pmod 5$, $i = 1 \pmod 5$; k is even

$= 20k-1$, $i = 5k+1$; k is odd;

$= 20k+3$, $i = 5k+2$; $= 20k+5$, $i = 5k+3$; $= 20k+7$, $i = 5k+4$

$f(v_{5n+i}\, v_{(i-1)5+j}) = 7+2j+20(i-1)$, $i = 1$ to n, $j = 1$ to 5

$f(v_{5n+i}\, v_{5n+i+1}) = 21 + 20(i-1) = 20i + 1$; $i = 1$ to n ; i is odd; $= 20i-1$, $i = 1$ to n ; i is even.

Theorem 3.2: The connected graph $P_n * nF_7$ is edge-odd graceful.

Proof: One of the labelings for vertex set is mentioned below.

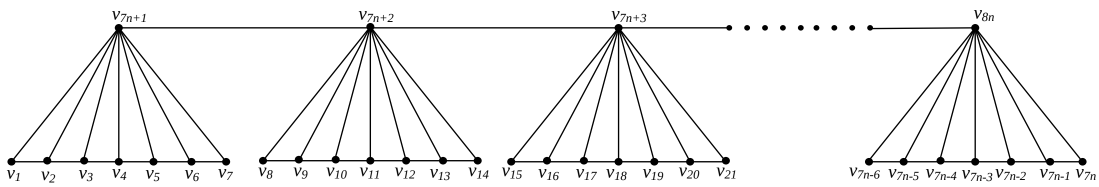

Figure.2 $P_n * nF_7$ graph

Define $f: E(G) \rightarrow \{0,1,2,\ldots,q\}$ by $f(v_i v_{i+1}) = 2i-1$, $i = 1, 2, 3, 4, 5, 6$.

$f(v_i v_{i+1}) = f(v_j v_{j+1}) + 28k$; $i = 7k+j$, $i = 7$ to $7n$, $i \neq 0 \pmod 7$; and $j = 1,2,3,4,5,6$.

$f(v_{7n+1}v_1) = 15$; $f(v_{7n+1}v_2) = 13$; $f(v_{7n+1}v_3) = 17$; $f(v_{7n+1}v_4) = 19$; $f(v_{7n+1}v_5) = 21$; $f(v_{7n+1}v_6) = 23$; $f(v_{7n+1}v_7) = 25$.

$f(v_{7n+i}v_{(i-1)7+j}) = f(v_{7n+1}v_j)+28k$; i = 2 to n , and j = 1 to 7

$f(v_{7n+i}v_{7n+i+1})= 27+(i-1)28$; i = 1 to (n-1).

Theorem 3.3: The connected graph $C_7 * 3F_n$ is edge-odd graceful.

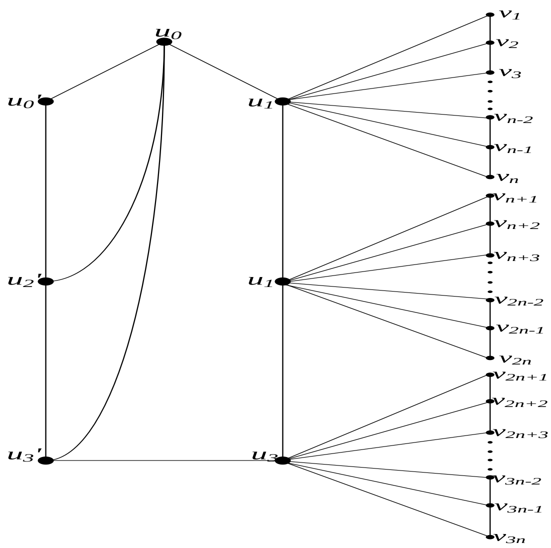

Figure.3 $C_7 * 3F_n$

One of the arbitrary labeling for vertices is given above in the figure.

Here p = 3n + 7, q = 6n + 6, 2q = 12n + 12.

Define f: E(G) → {1, 3, 5,….,2q-1} by $f(u_0u_1') = 1$; $f(u_1'u_2') = 3$; $f(u_0u_2') = 5$; $f(u_2'u_3') = 7$; $f(u_0u_3') = 9$; $f(u_3'u_3) = 11$; $f(u_3u_2) = 15$; $f(u_2u_1) = 13$; $f(u_0u_1) = 17$

$f(u_1v_i) = 12n+15- 4i$, i = 1 to n

$f(v_iu_{i+1}) = 12n+13- 4i$, i = 1 to (n-1)

$f(u_2v_{n+i}) = 10n+7- 4i$, i = 1 to n

$f(v_{n+i}v_{n+i+1}) = 10n+5-4i$, i = 1 to (n-1)

$f(u_3v_{2n+i}) = 8n+1$ i = 1 to n

$f(v_{2n+i}v_{2n+i+1}) = 8n-3-4i$ $\qquad$ $i = 1$ to $(n-1)$.

Theorem 3.4: $P_n + \bar{K}2$ is edge odd graceful when n is odd.

Proof: One of the labelings for vertex set is as follows:

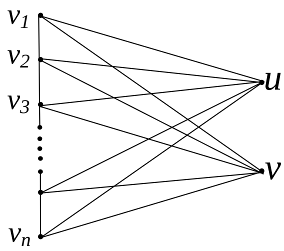

Figure.4 $P_n + \bar{K}2$

Define $f(uv_i) = 6i-5$, $i = 1$ to n; $f(vv_i) = 6i-3$, $i = 1$ to n; $f(v_iv_{i+1}) = 6i-1$, $i = 1$ to (n-1)

Its induced maps $f^+ : V(G) \rightarrow \{0,1,2,...,2q-1\}$ is defined as $f^+(V) = \sum_{u \in V} f(vu) (\bmod 2q)$. The resulting labeling for all vertices of G are all distinct. Therefore G is edge-odd graceful.

Example 3.5: $P_9 + \bar{K}2$ is edge odd graceful.

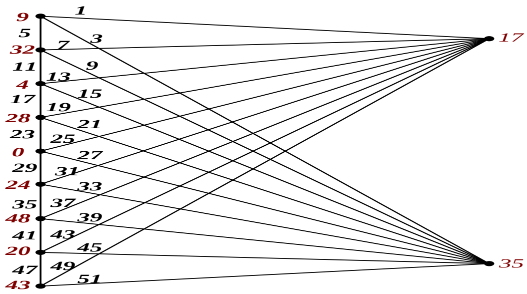

Figure.5 $P_9 + \bar{K}2$

Theorem 3.6: $C_n + \bar{K}2$ is edge-odd graceful. (n is odd)

Proof: One of the labelings for vertex set is as follows:

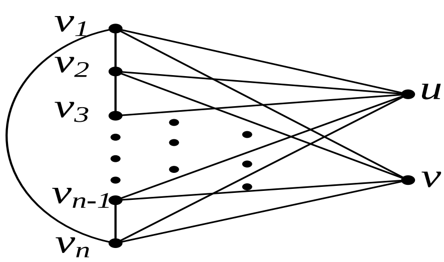

Figure.6 $C_n + \bar{K}2$

Define a map $f : V(G) \rightarrow \{1, 3, \ldots..\}$ by $f(uv_i) = 6i - 5$, $i = 1$ to n; $f(vv_i) = 6i - 3$, $i = 1$ to n ; $f(v_i v_{i+1}) = 6i - 1$, $i = 1$ to (n-1); $f(v_1 v_n) = 6n-1$. Its induced maps $f^+ : V(G) \rightarrow \{0, 1, 2, \ldots, 2q-1\}$ is $f^+(V) = \sum_{u \in V} f(vu) (\bmod 2q)$ for all vertex v in V. The resulting labeling for all vertices of G are all distinct. Therefore $C_n + \bar{K}2$ is edge-odd graceful.

Example 3.7: $\mathbf{C_7} + \bar{K}2$ is edge odd graceful.

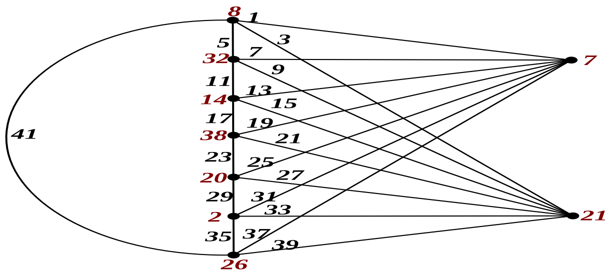

Figure.7 $\mathbf{C_7} + \bar{K}2$

4. PERFORMANCE EVALUATION

Theorem 4.1: $\mathbf{C_{n+k}} + \overline{K_2}$ is edge odd graceful when n is odd.

Proof: One of the labelings for vertex set is as follows:

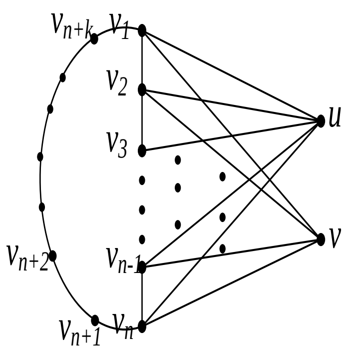

Figure.8 $\mathbf{C_{n+k}} + \overline{K_2}$

Define a map f : V(G) → {1, 3,…., (2q-1)} by $f(v_1v_n) = 6n-1$; $f(v_{n-1+i}v_{n+i}) = (6n-1) + 2(i-1) = 6n + 2i - 2$, i = 1 to (n+k-1); $f(v_{n+k}v_1) = (2q-1)$. Its induced maps $f^+ : V(G) \to \{0,1,2,...,2q-1\}$ is defined by $f^+(V) = \sum_{u \in V} f(vu) (\mathrm{mod}\, 2q)$.

Hence it is observed that the resulting labeling for all vertices of G are all distinct. Therefore G is edge-odd graceful.

Example 4.2: $\mathbf{C}_{7+4} + \overline{K_2}$ is edge odd graceful.

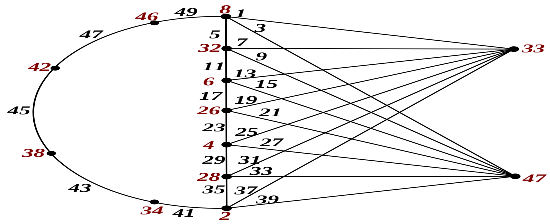

Figure.8 $\mathbf{C}_{7+4} + \overline{K_2}$

5. CONCLUSION

From the above theorems, we conclude that for $P_n * nF_5$, $P_n * nF_7$, $C_7 * 3F_n$, $P_n + \overline{K}2$, $C_n + \overline{K}2$, $\mathbf{C}_{n+k} + \overline{K_2}$ the labeling are proved as edge odd graceful.

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