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On achromatic Coloring of Central, Middle and Total graph of Bistar graph and Corona of some graphs and its Complement

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Abstract

An achromatic coloring of a graph G is a proper vertex coloring of G using k colors such that every pair of color classes is adjacent by at least one edge. The largest possible number k for which G admits such k colors is known as achro-matic number (G) of G. In this note, we present the achromatic number of Central, Middle and Total graph of Bistar graph $B_{m;n}$ for $m < n$, $m > n$ and $m = n$ respectively and also we obtain the Corona graph of Complete graph with its complement and Corona graph of Star graph with its complement.

Keywords : Achromatic coloring, Central graph, Middle graph, Total graph, Corona graph, Graph Complement

Mathematics subject classi cation: 05C15

1.Introduction

All graphs $G = (V; E)$ are assumed to be non- trivial, simple and undirected. A graph is a suitable way of representing information concerning relationship between objects. The objects are represented by vertices and relation by edges. A graph is typically denoted by G with vertex set $V = V(G)$ and edge set $E = E(G)$.

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The degree of a vertex v is denoted by $d(v)$. The minimum vertex degree is denoted by = (G), and the maximum vertex degree is denoted by $\Delta = \Delta(G)$. If G is a mapping f : v ! f1; 2; ::::kg admits k-colors then for any two adjacent vertices u and v, we have $f(u) \neq f(v)$ shows that f is a proper coloring.The chromatic number = (G) is the minimum k for which G has a proper k- coloring. An achromatic number of a graph G, represented by (G) is the largest number n such that G acknowledge complete n-coloring [4] which means that any graph G with n points has a n-coloring, but which is complete only for the complete graph K_n. A homomorphism of G onto G' is a function f from $V(G)$ onto $V(G')$ such that the points a and b are adjacent in G, then their images af and bf are adjacent in G'. This proves that f preserves adjacency [7].

2.Preliminaries

The Central graph [8,9,10] $C(G)$ of graph G is attained from G by subdividing every edge of G precisely once and then joining every point of vertices of the original graph which were previously non - adjacent.

The Middle graph [6] of G which is represented by $M(G)$. $M(G)$ has vertex set $V(G)$ [$E(G)$. Two vertices a,b of $M(G)$ are neighboring in $M(G)$ in case one of the following holds.

(i) a,b are within $E(G)$ and a,b are neighboring in G

(ii) a is within $V(G)$, b is within $E(G)$, and a,b are neighboring in G.

Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. Total graphs are generality of Line graphs. The Total graph of a graph G, represented by $T(G)$ is exempli ed as follows. The vertex set of $T(G)$ is $V(G)$ [$E(G)$. Two vertex a,b in the vertex set of $T(G)$ are neighboring in $T(G)$ in case one of the following holds.

(i) a,b are within $V(G)$ and a is neighboring to b in G.

(ii) a,b are within $E(G)$ and a,b are neighboring in G.

(iii) a is within $V(G)$, b is within $E(G)$ and a,b are incident in G.

The Bistar graph $B_{m;n}$ is the graph attained from K_2 by adhering m pendent edges to one end and n pendent edges to the other end of K_2.

Consider two graphs G_1 and G_2 on disjoint sets of k_1 and k_2 vertices respec-tively. The Corona $G_1 \circ G_2$ of G_1 and G_2 is classi ed as the graph attained by taking one copy of G_1 and k_1 copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2 [6].

The complement of a graph $G = (V; E)$ is denoted by $\overline{G}$ which is described as a simple graph having same vertex set as G and consider two vertices p and q are neighboring only when they are not neighboring in G [3]. For any graph G, the non-adjacent vertices are neighboring on its complement [11,12].

A simple graph in which every pair of individual vertices are neighboring in a complete graph and is denoted as K_n on n vertices.

A star S_k is the complete bipartite graph $K_{1;n}$; a tree with one internal node and k leaves.

In this paper, we nd the achromatic coloring for the Central graph, Middle graph and Total graph of Bistar graph, and also with the Corona graph of Star graph with its Complement and Corona graph of Complete graph with its Comple-ment which are denoted as $[C(B_{m;n})]$, $[M(B_{m;n})]$, $[T(B_{m;n})]$, $[K_{1;n} \circ \overline{K_{1;n}}]$ and $[K_n \circ \overline{K_n}]$ respectively.

3. Achromatic coloring of Central Graph of Bistar Graph Theorem 3.1.

The achromatic number for $C(B_{m;n})$ is $m + n + 2$, for $m; n$ 3.

ie., $[C(B_{m;n})] = \begin{cases} m + n + 2; & m < n \\ \vdots & \end{cases}$

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Proof

Let $B_{m;n}$ be the Bistar graph obtained from K_2 by joining m pendent edges to one end and n pendent edges to the other end of K_2. Here the graph G be a nite, undirected graph with no loops and multiple edges. Let $V(B_{m;n}) = fv_i; u_j ; : 1\ i\ 2; 1\ j\ m + ng$ and $E(B_{m;n}) = fv_1; u_j ; v : 1\ j\ m\ and\ v_2; u_j : 1\ j\ ng$. The graph $C(G)$ of the graph G is obtained by subdividing each edge of G exactly once and joining all the non-adjacent vertices of G. Let $w_{i;j}$, i = 1,2 and j= 1,2,.....n be the newly introduced vertex in the
edge connecting v_i and u_j and v_{12} is another newly introduced vertex between the vertices v_1 and v_2. The vertex set of $C(B_{m;n})$ is de ned as follows :

ie.,$V[C(B_{m;n})] = fv_i; u_j ; w_{ij1} ; w_{ij2} ; v_{12} : 1\ i\ 2; 1\ j\ m + n; 1\ j_1\ m; 1\ j_2\ n$ for $i = 1; 2g$.

Here v_i and u_j forms a clique of order $m + n + 1$.ie., v_i for $1\ i\ 2$ forms a clique of order $m + n + 1$, also u_j for $1\ j\ n$ forms another clique.

In $C(B_{m;n})$, we observe that v_1 is adjacent with u_j , for $m + 1\quad j\quad n$ and not adjacent with v_2 and u_j for $m + 1\quad j\quad m$. Similarly v_2 is adjacent with u_j for $1\quad j\quad m$ and not adjacent with u_j for $m + 1\quad j\quad n$.

Consider the color class C $=fc_1; c_2; :::::::; c_{m+n+2}g$. Now assign a proper coloring to these vertices to make the coloring as achromatic. To get the maximum number of pair of colors assign the color c_j to u_j for $1\ j\ m + n$. Assign the color c_1 to v_{12}, and assign the color c_{m+n+1} to v_i for $1\ i\ 2$. For $1\ j_1\ m + n$. Assign the color c_{m+n+2} for w_{1j1} and for $1\ j_2\ n$, assign the color c_{m+n+2} for w_{2j2}.

To prove the above said coloring is achromatic, consider any pair $(c_i; c_j)$.

Case 1

If $j = i + 1$, then the edge joining the vertices u_{j1} and u_j will stand for the pair $(c_1; c_j)$ for $1\ i\ m + n\ 1$.

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If $j = i + 1$ then the edge joining the vertices for u_2 and for u_j will stand for the pair $(c_2; c_j)$ for 2 i $m + n$ 1.

If $j = i + 1$ then the joining the vertices $u_{m+n\ 1}$ and u_j will stand for the pair $(c_{m+n\ 1}; c_j)$ for i = m+n-1.

Case 2

If $i = 1; 2; :::::m$ then the edge joining the vertices u_i and v_2 will stand for the pair $(c_i; c_{m+n+1})$.

If $i = m + 1; m + 2; :::::::::m + n$ then the edge joining the vertices u_i and v_1 will stand for the pair $(c_i; c_{m+n+1})$.

Case 3

If $i = 1$ and $j = 1; 2; ::::m$, then the edge joining the vertices u_j and w_{ij} will stand for the pair $(c_j ; c_{m+n+2})$.

If $i = 2$ and $j = 1; 2; ::::m$ then the edge joining the vertices for v_i and w_{ij} will stand for the pair $(c_{m+j} ; c_{m+n+2})$.

If i =1 and $j = 1; 2; ::::m$ then the edge joining the vertices v_i and w_{ij} will stand for the pair $(c_{m+n+1}; c_{m+n+2})$.

Thus the given coloring is achromatic and by the very construction, it is the maximal color class.

Therefore $[C(B_{m;n})] =$ 8 > > $m + n + 2;$ $m < n$ $m > n$ $m = n$:

4. Achromatic coloring of Middle Graph of Bistar Graph Theorem 4.1.

For any bistar graph $B_{m;n}$, the achromatic number, 8

$[M(B_{m;n})] =$ > $m + 4$ *or* $n + 3$ for $m < n$ *and* $m > n$

:

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Proof

Consider the Bistar graph $B_{m;n}$. By the de nition of Bistar, let u_1, u_2,......., u_m be the m pendant edges fond of the vertex u and v_1, v_2,......., v_n be other n pendant edge fond of the vertex v.

By the de nition of middle graph, each edge of uu_i and vv_j, (1 i $m;$ 1 j n) of $B_{m;n}$ is subdivided by the vertices u_i, and v_j for 1 i $m;$ 1 j n for $B_{m;n}$.

Now the vertex set of $M(B_{m;n})$ can be written as,
$V[M(B_{m;n})]$ =fug [fvg [fu_i : 1 i mg [fv_j : 1 j ng [fwg [fu'_i : 1 i mg [fv_j' : 1 j ng.

In $M(B_{m;n})$, u'_1, u'_2,....... u'_m along with u and w induce a clique of order m+ 1. Also v_1', v_2',..... v_n' along with v and w induce another clique of order n + 1. From this we can assign c_{m+1} or c_{n+1} colors to the graph $M(B_{m;n})$, for $m < n$ and $m > n$. According to the achromatic coloring we maximize the colors which is assign to the vertices of the graph as m + 4 or n + 3.

Therefore [$M(B_{m;n})$] m + 4 or n + 3.

Now for assigning a proper coloring to $M(B_{m;n})$, consider the following cases:

Case 1 For $m < n$ and $m > n$

Consider the color class C = $fc_1; c_2; :::::::; c_{m+1}$ or $c_{n+3}g$. Assign the color c_1 to the common vertex w and assign the color c_{m+2} to u and c_2 to v. For 1 i m, assign the color c_{i+1} to u_i and for 1 i n, assign the color c_{i+3} to v_i. Finally assign the color $c_{m+n\ 1}$ to the vertices of u_i for 1 i m and to assign the color c_{m+5} to v_i for 1 i n. Thus these coloring accommodates all the vertices except one pair ($c_3; c_{m+4}$). Thus to hold the missing one we must assign the color as c_3 to the vertices v_i for 1 i n. Hence assigning the maximum number of coloring other than m+4 is not possible. This implies [$M(B_{m;n})$] m+4 is unattainable. Hence we have

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$[M(B_{m;n})]$= m+4 for $m < n$ and $m > n$.

Case 2 For $m = n$

Consider the color class C = $fc_1; c_2; ::::::::; c_{m+4}$ or $c_{n+4}g$. Assign the color c_1 to the common vertex w and assign the color c_{n+4} to the vertices u and v respectively. Now coloring the vertices u_i and v_i for 1 i m and 1 i n in clockwise direction. For 1 i m, assign the color c_{i+2} to the vertices $u_i$and for 1 i n 2, assign the color c_{i+3} to v_i and for i = n-1, assign c_{n+4} to v_i and assign the color c_{n+3} to v_n. Next assign the color c_{n+3} to u_1 and for the remaining vertices of u_i ie., for 2 i m, assign the color c_{n+4}. Similarly assigning the colors c_2 to v_i for 1 i n 1 and for i = n assign c_3 to v_i. In $M(B_{m;n})$ we have a clique of order m+1 and another clique of order n+1 so we can assign c_{m+1} and c_{n+1} colors to this graph. But according to the course of action of achromatic coloring we can assign maximum number of colors to the vertices of the graph $M(B_{m;n})$ for m = n. So we can widen c_{n+4} color to c_{n+5} color to the vertices of u_i for 2 i m. If we assign such a manner, we cannot hold one pair ($c_3; c_{n+4}$). This shows that c_{n+5} color is not possible to assign to the vertices. Thus we have

$[M(B_{m;n})]$ $m + 4$ or $n + 4$ for $m = n$.

Therefore, $[M(B_{m;n})]$8= $n + 4$ for $m = n$.

Hence $[M(B_{m;n})]$ = > $m + 4$ *or* $n + 3$ for $m < n$ *and* $m > n$

$m = n$

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5. Achromatic number of Total Graph of Bistar Graph Theorem 5.1.

For any Bistar graph $B_{m;n}$, the achromatic number,

$$[T(B_{m;n})] = \begin{cases} m+5 \; or \; n+4 & \text{for } m<n \\ m+4 \; or \; n+5 & \text{for } m>n \\ m+5 \; or \; n+5 & \text{for } m=n \end{cases}$$

:

Proof

Let $V(B_{m;n}) = fu; v; u_1; u_2; ::::::; u_m; v_1; v_2; :::::::; v_n g$ and $E(B_{m;n}) = fw; u'_1; u'_2; ::::::::; u'_m; v_1'; v_2'; ::::::::; v_n' g$. By the de nition of total graph, we have $V[T(B_{m;n})] = fug \,[\, fvg \,[\, fu_i : 1 \quad i \quad mg \,[\, fv_i : 1 \quad i \quad ng \,[\, fwg \,[\, fu'_i : 1 \quad i \quad mg \,[\, fv_i' : 1 \quad i \quad ng$ in which the vertices u'_1, u'_2,....., u'_malong with u and w induce a clique of order $m + 1$. Also v_1' , v_2' ,........,v_n' along with v and w induce another clique of order $n + 1$. Now we can assign c_{m+1} or c_{n+1} color to the graph $T(B_{m;n})$, for $m < n;\ m > n$ and $m = n$. But according to the de nition of achromatic coloring, there is a maximum coloring possibility to the vertices of the graph $T(B_{m;n})$ as m+5 or n+4. Therefore we can assume that $[T(B_{m;n})]$ $m + 5$ or $n + 4$.

Consider the color class C = $fc_1; c_2; :::::::; c_{m+5}$ or $c_{n+4}g$. To determine the proper coloring, consider the following cases and to make the coloring as achromatic one.

Case 1 For $m < n$

Assign the color c_1 to the vertex w and assign the colors c_{m+4} and c_{n+4} to the vertices u and v respectively. Here we assign the colors in clockwise direction. Now assign, c_{i+1} to the vertices of u'_i for 1 i m and assign c_{m+2} to u_1 and c_{m+3} to u_i for 2 i m. For 1 i n, assign the color c_{i+2} to the vertices v_i' and assign the color c_{n+3} to the vertices of v_i for 1 i n 1 and for v_n assign c_2. To get a maximum number of pairs of colors assign the colors c_{m+5} and c_{n+5}

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to the vertices u and v. Here we fail to spot the pairs $(c_i; c_{n+3})$, $(c_{n+2}; c_{n+3})$ and $(c_{n+3}; c_{n+4})$, i=1,2. Thus the coloring of u and v should be in such a mode that it should hold all the above mentioned pairs. That is, here maximization is not probable.

Hence we have, $[T(B_{m;n})]$ $m + 5$ or $n + 4$.

Therefore from 1., we have $[T(B_{m;n})] = m + 5$ or $n + 4$.

Case 2 For $m > n$

Assign the color c_1 to the vertex w and assign the colors c_{m+2} and c_{n+4} to the vertices of u and v. Now assigning the colors in clockwise direction. Assign the color c_{i+1} to the vertices of u'_i for 1 i m and assign c_{m+4} to $u_i$for 1 i m 1 and for u_m, assign the color c_{m+3}. Now for 1 i n 1, assign the color c_{i+2} to the vertices of v_i' and for v_n', assign the color c_{n+5}. Finally assign the color c_2 to v_1' and c_{n+2} to v_n' v and for 2 i n 1, assign the color c_{n+5} to all v_i'. If we introduce one more color c_{m+5} or c_{n+6} to the vertices of u_i for i = 1,2,......n-1, then we replace the color c_{m+4} or c_{n+5} by c_{m+5} or c_{n+6}, then this recoloring will avoid the pairs $(c_i; c_{m+5})$, $(c_{i+1}; c_{m+3})$, $(c_{i+1}; c_{m+5})$, $(c_{m+1}; c_{m+4})$, $(c_{m+1}; c_{m+5})$ and $(c_{m+2}; c_{m+5})$, i = 1. This shows that, the new coloring is no more achromatic. Hence the coloring is maximum and we have, $[T(B_{m;n})]$ $m + 4$ or $n + 5$.

Therefore $[T(B_{m;n})] = m + 4$ or $n + 5$.

Case 3 For $m = n$

Assign the color c_1 to the vertex w and assign the colors c_{m+3} and c_{n+4} to the vertices of u and v. Assign the color c_{i+1} to the vertices of u'_i for 1 i m 1 and assign c_{m+2} to the vertex u'_m. Assign the colors c_{m+5} and c_{m+1} to the vertices of u_1 and u_m respectively and for the remaining vertices of u_i, that is 2 i m 1, assign c_{m+3}. Next assign the color c_{i+2} to the vertices of v_i' for 1 i n 1 and assign c_{n+5} to v_n'. Also assign c_{n+2} to v_i for 1 i n except i = n-1. For $v_{n\ 1}$, assign the color c_2. Here the vertices u_1', u_2',......u_m' and v_1', v_2',......v_n' forms

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a complete graph of order n. If we assign c_{m+6} or c_{n+6} colors to the vertices of $T(B_{m;n})$, we will fail to spot some pairs in $(c_i; c_{m+6})$ or $(c_i; c_{n+6})$ for 1 i m or n. So this contradicts the de nition. Hence we conclude that c_{m+5} or c_{n+5} is the maximum number of coloring. This shows that, in this case for m = n, we have

$[T(B_{m;n})]$ $m + 5$ or $n + 5$.

Hence from the comparison of 1, nally we have for $[T(B_{m;n})] = m + 5$ or $n + 5$.

Therefore $[T(B_{m;n})]$= $m + 5$ or $n + 4$ for $m < n$; $m + 4$ or $n + 5$ for $m > n$; $m + 5$ or $n + 5$ for $m = n$

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6. Achromatic number for Corona graph of some graphs and its com-plement

Theorem 6.1.

The achromatic number for Corona of Star graph and its complement is $2n+1$ ie., $[K_{1;n} \circ \overline{K_{1;n}}] = 2n + 1$

Proof

Let $V(K_{1;n}) = fv; v_i : 1$ i ng

Let $V(\overline{K_{1;n}}) = fv' ; u^k; v_i' ; u^k_i : 1$ i $n; 1$ k ng

By the de nition of Corona graph, each vertex of $K_{1;n}$ is adjacent to every vertex of a copy of complement of $\overline{K_{1;n}}$ which says that every vertex of $V(K_{1;n})$ is adjacent to every vertex from the set $V(\overline{K_{1;n}})$. Thus the corona of two graphs can be de ned as, $V([K_{1;n} \circ \overline{K_{1;n}}]) = fv; v_i : 1\ i\ ng\ [\ fv'\ u^k : 1\ k\ ng\ [\ fv_i'\ u^k_i : 1\ i\ n; 1\ k\ ng.$

Let $K_1{}^{(1)}{}_{;n}$, $K_1{}^{(2)}{}_{;n}$,........, $K_1{}^{(n)}{}_{;n}$ be the n-copies of the complement of the Star graph. Here u^k and also u^k_i induce a clique of order n. Along with this reason, we can assign c_n colors to the corona graph $[K_{1;n} \circ \overline{K_{1;n}}]$, for any positive integer n 3. But from the de nition of achromatic coloring we have to maximize the colors

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which is assigned to the vertices of this graph as c_{2n+1}. Therefore we can say that

$[K_{1;n} \circ \overline{K_{1;n}}] = 2n + 1$.

Now assigning a proper coloring to the vertices of $[K_{1;n} \circ \overline{K_{1;n}}]$. Consider the color class C = $fc_1; c_2; :::::::; c_{2n+1}g$

◦ Assign the color c_1 to v and c_{i+1} to v_i for 1 i n.

◦ Assign the color c_{2n+1} to v_i' for 1 i n 1 and assign c_2 to v' and v_n'.

◦ Assign the color c_{n+i} for 2 i n + 1 to u_k. ◦

Assign the color c_{i+2} to u^1_i for 1 i n.

◦ Assign, c_{n+j} to u^k_i for 2 k n, 1 i n 1 and $i = 3$ with 2 j n and for u^k_3 where $k = 2; 3$ assign the color c_2 and c_3. For the remaining vertices of u^k_3 ie., for 4 k n 1,allot c_{k+2} and for the vertex u^n_3, allot,$c_{2n\ 2}$.

◦ Assign the color c_{n+i} for 3 k $n + 1$.

If we can introduce c_{2n+2} to the vertices of v_i' for 1 i n 1, then it will avoid the pairs $(c_i; c_{2n+1})$,2 i n 1 to locate the new coloring. This means that introducing new coloring is not possible. So we can say that, $[K_{1;n} \circ \overline{K_{1;n}}]$ 2n + 1.

But from the assumption, we have $[K_{1;n} \circ \overline{K_{1;n}}] = 2n + 1$., for any positive integer n 3.

Theorem 6.2.

The achromatic number for Corona of Complete graph and its complement is n + 1

ie., $[K_n \circ \overline{K_n}] = n + 1$, for any positive integer n.

Proof

Let v_1,v_2,......,v_n be the vertices of Complete graph and u_{11},u_{12},.....,u_{1n}, u_{21},u_{22} u_{23},.....,u_{2n},......,u_{n1},u_{n2},......,u_{nn} be the vertices of complement of the Complete graph K_n.

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ie., $V(K_n) = \{v_1; v_2; \ldots; v_n\}$ and $V(\overline{K_n}) = \{u_{11}; u_{12}; \ldots; u_{1n}; u_{21}; u_{22}; \ldots; u_{2n}; \ldots; u_{n1}; u_{n2}; \ldots; u_{nn}\}$.

By applying the de nition of Corona graph , every vertex of K_n is adjacent to every vertex of a copy of a complement graph of a Complete graph, ie., each vertex $v_i \in V(K_n)$ is adjacent to each vertex from the set $\{u_{ij} : 1\ i\ n; 1\ j\ n\}$. Thus the corona of two graphs, Complete graph and its complement graph is,

$V(K_n \circ \overline{K_n}) = \{v_i : 1 \quad i \quad n\} \cup \{u_{ij} : 1 \quad i \quad n; 1 \quad j \quad n\}$.

Consider the color class C = $\{c_1; c_2; \ldots; c_{n+1}\}$. Assign the following $n + 1$ colors to make the coloring as achromatic one. Assign the color c_i to the vertices v_i for $1\ i\ n$ in clockwise direction and assign c_{n+1} to u_{ij} for $1\ i\ n; 1\ j\ n$. An easy check shows that this coloring accommodate all the pairs of color class. Therefore the coloring is achromatic one and hence which is maximum.

Hence we have $[K_n \circ \overline{K_n}] = n + 1$, for any positive integer n.

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